MATH 3B3 (Fall 2016)
Assignment 1

Due Date: Before 12:30 pm, September 23, 2016

1. Let $\gamma(t) : [a, b] \longrightarrow \mathbb{R}^3$ be a C^1 curve. Show that its arclength is independent of reparametrization. [Hint: Use the fact explained in class that a reparametrization is either monotonically increasing or decreasing. In each case use the formula for integration by substitution.]

2. For the curve $\mathbf{c}(t) = (e^t \cos t, e^t \sin t, e^t)$ with $t > 0$, find its reparametrization by arclength.

3. Verify that the curve $\mathbf{c}(s) = \frac{1}{\sqrt{5}}(\sqrt{1+s^2}, 2s, \log(s + \sqrt{1+s^2}))$ has a unit speed parametrization. Then compute its curvature and torsion. In this course, log always refers to the logarithm base e.

4. A regular curve $\mathbf{c}(t)$ has the property that there is a vector $\mathbf{a} \in \mathbb{R}^3$ such that $\mathbf{c}(t) - \mathbf{a}$ is always perpendicular to its tangent vector $\mathbf{c}'(t)$. Prove that $\mathbf{c}(t)$ must lie on a sphere.

5. Show that the formula

$$\phi(u, v) = (u \cos v, u \sin v, u + v)$$

for $u, v \in \mathbb{R}$ gives a parametrization of a regular smooth surface. Find the equation of the tangent plane and the unit normal vector to the surface at the point $(u, v) = (a, b)$.