Homework 2

Name: ____ Math 2210Q, Fall 2016

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Due: Monday, September 19

1. Given the matrices

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & 5 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -3 & 0 \\ 4 & 1 \\ 5 & -2 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 0 & -5 \\ 3 & 0 \end{bmatrix}$$

calculate each of the following if it exists:

$$\mathbf{AB}+\mathbf{C}, \quad \mathbf{BA}+\mathbf{C}, \quad \mathbf{A}(\mathbf{B}+\mathbf{C})$$
$$\mathbf{B}(\mathbf{C}+\mathbf{D}), \quad \mathbf{BC}+\mathbf{D}, \quad \mathbf{ABC}$$
$$\mathbf{CD}, \quad \mathbf{DC}, \quad \mathbf{CA}, \quad -2\mathbf{D}$$

2. Calculate the product

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Use this example to show that given $\mathbf{AB} = \mathbf{AC}$ we cannot conclude $\mathbf{B} = \mathbf{C}$ (this is known as the law of cancellation, and it is true for real numbers but not for matrices).

3. Calculate the product

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

and find the value of b such that the result is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Find a dependence relation between the columns of

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & 5 \end{bmatrix}$$

Is any column a multiple of another column?

5. Find a dependence relation between the rows of

$$A = \begin{bmatrix} 21 & -28 & 35 \\ 3 & -4 & 5 \end{bmatrix}$$

6. Find a linearly independent subset of the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

Can each of the four vectors be written as a linear combination of the other three?

7. Suppose a linear system has the solution

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

a. What is the dimension of the span?

b. What is a parameterization of the x, y, z coordinates of a vector in the span?

c. Is the vector $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ in the span?

d. Is the solution set

  (a) the same as the span,
  (b) parallel to the span,
  (c) perpendicular to the span,
  (d) none of these.

8. True or False?

a. If 3 vectors lie in the same plane, then there is a dependence relation between them.

b. If a set of vectors is linearly dependent, then a vector in the set is a scalar multiple of one of the others.

c. If a set of vectors is linearly dependent, then each vector in the set can be written as a linear combination of the others.

d. If a set of vectors is linearly dependent, then there exists a vector in the set that can be written as a linear combination of the others.

e. The columns of any 4×5 matrix are linearly dependent.

f. A set of fewer than n vectors in $\mathbb{R}^n$ is linearly independent.