Name:

Instructions: Staple all work to the back of this page.

Problem 1: Create truth tables for the following statements.

a. $(p \vee q) \wedge \sim p$

b. $\sim (p \wedge q) \vee p$

c. $(p \vee q) \wedge (q \vee r)$

Problem 2: Show that the following are logically equivalent by means of a truth table:

a. $p \vee (q \vee r) \equiv (p \vee q) \vee r$ (i.e. The Associative Law Holds)

b. $\sim (p \wedge q) \wedge p \equiv \sim q \wedge p$

c. $\sim (p \vee (\sim q \wedge \sim r)) \equiv \sim p \wedge (q \vee r)$

Problem 3: Use truth tables to determine which of the following are tautologies and which are contradictions.

a. $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$

b. $(p \wedge \sim q) \wedge (\sim p \vee q)$

c. $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

d. $(\sim p \vee q) \vee (p \wedge \sim q)$

Problem 4: (The Definition of Exclusive-Or) Let $p \oplus q$ be shorthand for the statement $(p \vee q) \wedge \sim (p \wedge q)$.

a. By writing a truth table for $(p \vee q) \wedge \sim (p \wedge q)$ verify that:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

You may view the truth table above as the definition of $p \oplus q$; this is the notion of "exclusive-or": p or q is true but not both. It may be easier to think of $p \oplus q$ as being true when exactly one of p or q is true. Answer the following questions about the algebraic structure of exclusive-or.

b. Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? (i.e. Is $\oplus$ associative?) Justify by a truth table if it is true, or find specific truth values for p, q, and r which would show that they aren't equivalent.

c. Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? (i.e. Does $\wedge$ distribute over $\oplus$?) Justify by a truth table if it is true, or find specific truth values for p, q and r which would show that they aren't equivalent.