

- 1, Given: A is an open subset of R^d , $A \cap \overline{B} \neq \emptyset$. Show: $A \cap B \neq \emptyset$.
2. Given: U is an open subset of R^d
 - a) Show: $U \subseteq (\overline{U})^\circ$
 - b) Can we have $U \neq (\overline{U})^\circ$?
3. Given: A is a subset of R^d . Show: $A' = (\overline{A})'$ (A' = set of all limit points of A)
4. For $A \subseteq R^d$, diameter of A , $\text{diam}(A) = \sup \{\|x - y\| : x, y \in A\}$
 - a) Show that $\text{diam}(A) < \infty$ if and only if, A is bounded.
 - b) Show that if A is compact then there exist $x_0, y_0 \in A$ such that $\text{diam}(A) = \|x_0 - y_0\|$
5. Given a sequence $\{X_n\}$ in R^d , $x \in R^d$, consider the statements
 - a) given any nbd (=neighborhood) U of x , $\{n : X_n \in U\}$ is infinite.
 - b) given any nbd U of x , $\{X_n\}$ is frequently in U (for every $n \in \mathbb{N}$, there exists $k > n$, such that $X_k \in U$).
 - c) there is a subsequence of $\{X_n\}$ that converges to x .
 show: 1): $a \Rightarrow b$
 2): $b \Rightarrow c$
 3): $c \Rightarrow a$