

1. Given: A is an open subset of  $R^d$ ,  $A \cap \bar{B} \neq \emptyset$ . Show:  $A \cap B \neq \emptyset$ .

2. Given: U is an open subset of  $R^d$

a) Show:  $U \subseteq (\bar{U})^\circ$

b) Can we have  $U \neq (\bar{U})^\circ$ ?

3. Given: A is a subset of  $R^d$ . Show:  $A' = (\bar{A})'$  ( $A'$  = set of all limit points of A)

4. For  $A \subseteq R^d$ , diameter of A,  $\text{diam}(A) = \sup \{ \|x - y\| : x, y \in A\}$

a) Show that  $\text{diam}(A) < \infty$  if and only if, A is bounded.

b) Show that if A is compact then there exist  $x_0, y_0 \in A$  such that  $\text{diam}(A) = \|x_0 - y_0\|$

5. Given a sequence  $\{X_n\}$  in  $R^d$ ,  $x \in R^d$ , consider the statements

a) given any nbd (=neighborhood) U of x,  $\{n : X_n \in U\}$  is infinite.

b) given any nbd U of x,  $\{X_n\}$  is frequently in U (for every  $n \in N$ , there exists  $k > n$ , such that  $X_k \in U$ ).

c) there is a subsequence of  $\{X_n\}$  that converges to X.

Show: 1):  $a \Rightarrow b$

2):  $b \Rightarrow c$

3):  $c \Rightarrow a$