Math 3000 - Assignment 1 : Due Friday Sept 2 in class (15 pts)

NAME:

1. (4 pts) Let P, Q and R be statements. Is the following statement a tautology? (i.e. is it always true?)

$$\neg[R \Rightarrow (P \vee Q)] \Rightarrow [(R \wedge \neg P) \wedge \neg Q]$$

Justify your answer by completing the given truth table.

P	Q	R	$\neg[R \Rightarrow (P \vee Q)]$	$[(R \wedge \neg P) \wedge \neg Q]$	$\neg[R \Rightarrow (P \vee Q)] \Rightarrow [(R \wedge \neg P) \wedge \neg Q]$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

2. (4 pts) Use logical equivalences to show that $R \Rightarrow \neg P$ is equivalent to $(P \wedge R) \Rightarrow (R \wedge \neg R)$ (justify using the equivalences from class notes).

3. (3 pts) Consider the statement "For any real numbers x and y satisfying $x < y$, there exists a rational number q such that $x \leq q < y$.

   a. Express the statement in symbols using quantifiers.

b. Express the negation of the statement in symbols using quantifiers.

c. Write out the negation of the sentence in words. Points will be awarded for clarity and quality (no points for "It is false that ...").

4. (4 pts) Determine whether or not the following statements are true or false. Justify your answers by providing a proof (e.g. by direct proof or by contradiction) or by giving a counter-example.

   a. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z} : x - y = 2$

   b. $\exists x \in \mathbb{Z} \ : \ \forall y \in \mathbb{Z}, x - y = 2$

   c. $\exists y \in \mathbb{N} \ : \ \forall x \in \mathbb{R}, 2x^2 - 4x + 1 > y$

   d. $\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \exists z \in \mathbb{Z} \ : \ x + z = y - z$