MATH 179 – Introductory Business Mathematics – Tutorial Sheet 5 (selected)

1. Factoring polynomials

Factor the following polynomials by grouping

$$(i)\ 2x + 2y + ax + ay, \qquad (ii)\ 7r + 7s - kr - ks, \qquad (iii)\ xr + xs + yr + ys,$$

$$(iv)\ 2ax + 2bx + 3a + 3b, \qquad (v)\ 2xy + y^2 - 2x - y, \qquad (vi)\ 2a^4 + 2a^3 - 4a - 4.$$

2. Factoring trinomials

Factor the following trinomials, if possible.

$$(i)\ x^2 + 12x + 11, \qquad (ii)\ x^2 - 7x + 10, \qquad (iii)\ x^2 - 5x - 50, \qquad (iv)\ x^2 - 2x + 4,$$

3. Functions

Decide whether each of the equations below determines y as a function of x (solve for y first if necessary). Give the domain.

$$(i)\ y = x^2 \qquad (ii)\ x = y^2 \qquad (iii)\ y = 2x - 6$$

$$(iv)\ xy = 1 \qquad (v)\ y = \sqrt{4x + 2} \qquad (vi)\ y = \frac{2}{x-4}$$

4. Graphing Quadratics

Graph the following quadratics.

$$(i)\ x^2 + 12x + 11, \qquad (ii)\ -x^2 - 7x - 10, \qquad (iii)\ x^2 - 5x - 50, \qquad (iv)\ 2x^2 - 3x + 1$$

Identify if there exists a minimum or maximum, where it occurs and their value.

5. Functions

Given for $x > 0$, $\quad y = f(x) = \frac{2}{9-x^2} + \sqrt{x} - 3$ find

$$(i)\ f(2), \qquad (ii)\ f(x^2), \qquad (iii)\ f(x + a), \qquad (iv)\ f(2t)$$

6. Simplifying Logarithms and Exponentials

Using logarithm or index rules, simplify the following

$$(i)\ \ln 8 - \ln 2, \qquad (ii)\ 2\log_5 10 - 2\log_5 2 \qquad (iii)\ \frac{\log 100}{\log 10}$$

$$(iv)\ e^{2x}(e^{-3x} - e^x), \qquad (v)\ 2(e^x + e^{-x})^2 \qquad (vi)\ \left(\frac{e^{3x}}{e^{-7x}}\right)^3$$

7. Equations with variable exponents

Solve each equation for x

$$(i)\ 7^x = 5, \qquad (ii)\ 4^x = 3 \qquad (iii)\ 9^{-x+2} = 13 \qquad (iv)\ 3^{2x} = 14 \qquad (v)\ 2^{x+3} = 5^x$$

$$(vi)\ e^{0.006x} = 30 \qquad (vii)\ \log_{12} x = 0, \qquad (viii)\ \log_4 x = 0 \qquad (ix)\ \log_4 \sqrt{64} = x \qquad (x)\ \log_4(2x + 4) = 3$$

8. Arithmetic sequences

For each of the following sequences, say whether or not it is arithmetic. If it is, give the common difference and a formula for the $i^{\text{th.}}$ term

(i) $2, 5, 8, 11, 14, \ldots,$ $\qquad$ (ii) $2, -4, 6, -8, 10, -12, \ldots$ $\qquad$ (iii) $1, 2, 4, 7, 11, 16, \ldots$
(iv) $-6, -10, -14, -18, -22, \ldots$ $\quad$ (v) $3, \frac{15}{4}, \frac{9}{2}, \frac{21}{4}, 6, \ldots,$ $\qquad$ (vi) $1, \frac{1}{2}, 0, -\frac{1}{2}, -1, \ldots$

9. Summing arithmetic sequences
Compute the following sums

(i) $\sum_{i=1}^{10}(8i - 5),$ $\quad$ (ii) $\sum_{i=1}^{17}(i - 1),$ $\quad$ (iii) $\sum_{i=1}^{10}\left(\frac{1}{2}i - 1\right),$ $\quad$ (iv) $\sum_{i=1}^{15} 2i$

(v) $\sum_{i=1}^{20}(2i - 5),$ $\quad$ (vi) $\sum_{i=1}^{6}(3 - 5i),$ $\quad$ (vii) $\sum_{i=1}^{7}\left(4 - \frac{1}{3}i\right),$ $\quad$ $(viii)$ $\sum_{i=1}^{27}(-i)$

10. Word problems
(i) A machine purchased for business use depreciates, or loses value over a period of years. The value of the machine at the end of its useful life is called is scrap value. By one method of depreciation the scrap value S is given by
$$S = C(1 - r)^n$$
where C is the original cost, n is the useful life in years and r is the constant percent of depreciation.
(a) Find the scrap value of a machine costing \$30000, having a useful life of 12 years and a constant annual rate of depreciation of 15%.
(b) Suppose that a machine has a scrap value of half of its original cost after 6 years. Find the constant rate of depreciation.

(ii) Let the number of bacteria present in a certain culture be given by $B(t) = 25000e^{0.2t}$, where t is time measured in hours, and $t = 0$ corresponds to noon. Find, to the nearest hundred the number of bacteria present at

$\qquad$ **(a)** noon $\qquad$ **(b)** 1pm $\qquad$ **(c)** 2pm.

When will the population double?

(iii) A contaminent is being flushed out of a system. The amount A of contaminent left after time t hours is given by
$$A(t) = 3.25e^{-0.43t}.$$
How much contaminent was there originally? How much will be left after one hour? How long will it take for $\frac{3}{4}$ of the contaminent to be flushed out?

(iv) A rich uncle deposits \$1 in Mike's bank account on the first day of his birthday month, \$2 on the second day, \$3 on the third day, and so on for 30 days. How much will this amount to over the entire month?

(v) Jenni is offered a job at \$1600 per month with a guaranteed increase of \$50 every 6 months for 5 years. What will her salary be at the end of that time?

(vi) A stack of wood has 28 pieces on the bottom, 24 on top of those, then 20 and so on. If there are 108 pieces of wood how many rows are there?