MATH3403 Assigment 1

This assignment is worth 4% of the total assessment. Late penalties are listed in the profile. If you hand it in late, you must hand it to the assignment box (Level 4) AND email your tutor saying that you have done so.

Each problem is going to be marked from 0-10 with 6 points going to the procedure (including neatness and clarity) and 4 for the final answer. The total score for the assignment is 30 points.

1. Solve the equation $au_x + bu_y + u = 0$ with Cauchy data $u(s) = e^{-s}$ along the straight lines $(x(s), y(s)) = (cs, ds)$. The solution breaks down for a particular combination of a, b, c and d. Describe the combination geometrically and explain it in terms of characteristic curves.

2. Consider the equation $yu_x - xu_y = 0$ $(y > 0)$. Check for each of the following initial conditions wheter the problem is solvable. If it is solvable, find a solution, If it is not, explain why

   (a) $u(x, 0) = x^2$

   (b) $u(x, 0) = x$

3. Consider the equation $(1 - v^2)u_{xx} + 2(1 + v^2)u_{xy} + (1 - v^2)u_{yy} = 0$ where v is a constant real parameter with $|v| < 1$. After proving that this is an hyperbolic PDE show that its general solution (without Cauchy data) is similar to the general solution of the wave equation $(u_{xx} - v^2 u_{yy})$ with v equal to the propagation speed.