## MAT 2641: Real Analysis                    Assignment 1

**Note:**

• **Due Date:** By 10:30 AM on Friday 12 August 2016

**Question 1:**

Let $A$ and $B$ be subsets of the universal set $E$.

(a) Using set identities (in particular, using only equalities) simplify the following expressions
  (i) $A \setminus (A \setminus B)$,      (ii) $(A \cap B) \setminus (A \cap B^c)$.

(b) Prove by double inclusion the set identity $A \cap (A \cup B) = A$.

**[ 2, 2, 3 Marks]**

**Question 2:**

(a) Let $\mathbb{P}$ denote the set of positive numbers, which is assumed to exist. Reformulate the order axioms of $\mathbb{R}$ in terms of $\mathbb{P}$.

(b) Show that if $1 < a$, then $1 < a < a^2$.

**[ 3, 3 Marks]**

**Question 3:**

(a) Determine according to the values of the real number $x$ when the rational function $f(x) = \dfrac{x - 3}{6x^2 - x - 2}$ is positive, 0, or negative.

(b) Sketch the graph of $f \colon \mathbb{R} \to \mathbb{R} \colon x \mapsto |2x - 1| - |3x + 5| = f(x)$. For the sketch take $x$ between $-7$ and $2$, clearly indicating the $x$- and $y$-intercepts.

(c) Determine and sketch the set of pairs $(x, y)$ in $\mathbb{R} \times \mathbb{R}$ that satisfy $|x\,y| < 5$.

  (*Hint:* One may evaluate $|x\,y|$ according to the sign of $x\,y$. This gives rise to four cases. By letting $y$ be a function of $x$, the four cases correspond to two graphs. Clearly plot these graphs for $-10 \le x \le 10$, indicating only the relevant values for $x$. Then shade the required area and describe it explicitly.)

**[ 4, 4, 4 Marks]**