MATH 254, Summer 2016

Practice Questions for the Final Exam

Question 1: Determine the imaginary part of $(1-i)^{20}$.

Question 2: Evaluate $Im(\frac{2+i}{2-i})$.

Question 3: Find all solutions for the equation $z^3 = -27i$. Sketch the solutions in the complex plane.

Question 4: Determine if $f(z) = x^3 - 3xy^2 - i(y^3 - 3x^2y)$ is analytic. If yes, find its derivative.

Question 5: Determine if $f(z) = Im(z) + (z^*)^2$ is analytic. If yes, find its derivative.

Question 6: Find the Laurent series for $f(z) = \frac{1+z}{z(1-z)}$ valid for $|z| > 1$.

Question 7: Evaluate the integral $\int_0^{2\pi} \frac{dx}{3+\sin x}$ using the residue theorem. Explain each step in your calculation.

Question 8: Evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{3+x^4}$ using the residue theorem. Explain each step in your calculation.

Question 9: Evaluate the integral $\int_{-\infty}^{\infty} \frac{x^2+4}{x^4+16} dx$. Explain each step in your calculation. [Note that the algebra in this problem gets a bit tedious at the end.]

Question 10: Evaluate the integral $\oint_C \frac{e^z}{z^2(z-i)(z-2i)} dz$, where C is the circle with radius 1.5.Explain each step in your calculation.

Question 11: Evaluate the integral $\int_0^{2\pi} \frac{\cos(3\theta)}{5-4\cos\theta} d\theta$.