1. HW 2

1. Suppose $\{v_1, v_2, v_3, v_4\}$ is a set of linearly dependent vectors.

a. Suppose we apply the elementary operation of adding k times v_1 to v_3. Show the resulting set: $\{v_1, v_2, kv_1 + v_3, v_4\}$ is still linearly dependent.

b. Suppose $k \neq 0$ and we perform the elementary operation of multiplying v_2 by k. Show the resulting set: $\{v_1, kv_2, v_3, v_4\}$ is still linearly dependent.

2a. Let $M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 3 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix}$.

Use column operations to get M as close to the identity matrix as possible. Then determine for which of the vectors $b_1 = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix}$, $b_3 = \begin{bmatrix} 2 \\ 4 \\ 3 \\ 0 \end{bmatrix}$, $b_4 = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 0 \end{bmatrix}$, the equation $Mx = b_i$ has a solution. (You don't need to solve for x.)

2b. Let $M = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$.

Use column operations to get M as close to the identity matrix as possible. Then determine for which of the vectors $b_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $b_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $b_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $b_4 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, the equation $Mx = b_i$ has a solution. (You don't need to solve for x.)

3. $\mathbb{R}^3$ is what we call the vector space of vectors of length 3 with real entries. Which of the following sets is a basis of $\mathbb{R}^3$?

a. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

b. $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

c. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

e. $\begin{bmatrix}3\\1\\6\end{bmatrix}, \begin{bmatrix}2\\9\\7\end{bmatrix}, \begin{bmatrix}13\\1\\3\end{bmatrix}, \begin{bmatrix}7\\13\\2\end{bmatrix}$

f. $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}0\\0\\3\end{bmatrix}$

g. $\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

4. Suppose M is upper diagonal.

a. What must you know about the entries of the diagonal itself in order to be sure the equation $Mx = b$ has a solution for all b.

b. If the entries on the diagonal DO NOT satisfy the condition you stated in part a), is it still possible for the equation $Mx = b$ to have a solution for all b?