**Math 636** **Final - Quiz Component**

**1.** The system of linear equations

$$\begin{aligned} 2x_2 - x_3 &= 3 \\ 2x_1 + x_2 + 2x_3 &= 1 \\ x_1 + x_2 + 3x_3 &= 2 \end{aligned}$$

(a) is consistent with a unique solution.
(b) is consistent with infinitely many solutions.
(c) is inconsistent.

**2.** Which of the sets of polynomials in $P_2(\mathbb{R})$ is linearly dependent?
(a) $\{1 + 2x + 2x^2, 1 + x + x^2, x^2 + 4x + 4\}$
(b) $\{2, 2 + 3x, 2 + 3x + 4x^2\}$
(c) $\{3 + 2x - 2x^2, 2 - 2x + 2x^2, 1 + 4x + 3x^2\}$
(d) $\{-1 + x + 3x^2, 2 + 4x^2, 6 - 3x - 3x^2\}$

**3.** Let $A$ be an invertible matrix. Which of the following is false?
(a) $A$ must be $n \times n$.
(b) $\text{Row}(A)^{\perp} = \text{Null}(A^T)$.
(c) $\det(A^{-1}) \neq \det(A)$.
(d) The columns of $A$ form a basis for $\mathbb{R}^n$.

**4.** Which of the following statements is true?
(a) If $A\vec{v} = \lambda\vec{v}$, then $\vec{v}$ is an eigenvector of $A$.
(b) If $A$ is $n \times n$ and $\text{rank}(A) < n$, then $A$ has a row of zeroes.
(c) If $P$ is an orthogonal matrix, then $P$ is invertible.
(d) If $\{\vec{v}_1, \ldots, \vec{v}_k\}$ is a linearly independent set of vectors in $\mathbb{R}^n$, then $A = \begin{bmatrix}\vec{v}_1, \ldots, \vec{v}_k\end{bmatrix}$ is invertible.

**5.** Let $\mathbb{V}$ be a finite dimensional vector space and let $\{\vec{v}_1, \ldots, \vec{v}_k\}$ be a linearly independent set in $\mathbb{V}$. Which of the following statements is false?
(a) $\dim \mathbb{V} \geq k$.
(b) $\text{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$ is a subspace of $\mathbb{V}$.
(c) Every basis for $\mathbb{V}$ contains the vectors $\vec{v}_1, \ldots, \vec{v}_k$.
(d) If $\dim \mathbb{V} = k$, then $\text{Span}\{\vec{v}_1, \ldots, \vec{v}_k\} = \mathbb{V}$.

**6.** What is the geometric multiplicity of the eigenvalue $\lambda = 2$ of $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$?

(a) $g_\lambda = 1$
(b) $g_\lambda = 2$
(c) $g_\lambda = 3$

**7.** Which of the following is the vector $\vec{x}$ that is closest to being a solution for the system

$$\begin{aligned} x_1 + x_2 &= -2 \\ -x_1 - 2x_2 &= 3 \\ 3x_1 - 2x_2 &= 2 \end{aligned}$$

(a) $\begin{bmatrix} -3/10 \\ -43/30 \end{bmatrix}$ (b) $\begin{bmatrix} 9/20 \\ 43/20 \end{bmatrix}$ (c) $\begin{bmatrix} -1/4 \\ -43/36 \end{bmatrix}$ (d) $\begin{bmatrix} 1/4 \\ 43/36 \end{bmatrix}$

**8.** Which of the following is the best fitting equation of the form $y = a + bx$ for the following data?

| | | | |
|---|---|---|---|
| x | −1 | 0 | 1 |
| y | 5 | 2 | −2 |

(a) $\frac{5}{3} - \frac{7}{2}x$ (b) $\frac{5}{3} + \frac{7}{2}x$ (c) $2 - 3x$ (d) $\frac{11}{6} - \frac{10}{3}x$

**9.** In $\mathbb{R}^4$ using the inner product $\langle \vec{x}, \vec{y} \rangle = x_1y_1 + 2x_2y_2 + 3x_3y_3 + x_4y_4$. the following four vectors are orthogonal:

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 5 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 0 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 6 \end{bmatrix}$$

Let $\mathbb{S} = \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$. The projection of $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ onto $\mathbb{S}$ is:

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 0 \\ 2 \\ 7 \end{bmatrix}$ (c) $\begin{bmatrix} 8/\sqrt{7} \\ 8/\sqrt{7} \\ 6/\sqrt{7} \\ 1/\sqrt{7} \end{bmatrix}$ (d) $\begin{bmatrix} 8/7 \\ 8/7 \\ 6/7 \\ 1/7 \end{bmatrix}$

**10.** Let $A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, A_3 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \in M_{2\times 2}(\mathbb{R})$ and define $\mathbb{S} = \text{Span}\{A_1, A_2, A_3\}$.
If $D = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$, then $\text{proj}_{\mathbb{S}}(D)$ is:

(a) $\begin{bmatrix} 7/6 & 1 \\ 0 & 5/6 \end{bmatrix}$ (b) $\begin{bmatrix} 2/3 & 0 \\ 2/3 & 2/3 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 2 \\ 4 & 4 \end{bmatrix}$

For questions 11 - 20, determine if the statement is True or False. You should make sure that you have a proof of each true statement and a counter example for each false statement.

**11.** If the reduced row echelon form of $(A - \lambda I)$ is $I$, then $\lambda$ is not an eigenvalue of $A$.
(a) True.
(b) False.

**12.** If a system of 3 linear equations in 4 unknowns is such that the coefficient matrix has a rank of 2, then the system has infinitely many solutions.
(a) True.
(b) False.

**13.** If $A$ is not diagonalizable, then $A$ is not invertible.
(a) True.
(b) False.

**14.** If $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^3$, then $\vec{x} = s\vec{v}_1 + t\vec{v}_2,\ s, t \in \mathbb{R}$ is a vector equation for a plane in $\mathbb{R}^3$.
(a) True.
(b) False.

**15.** If $A$ is a $3 \times 3$ matrix, and $\vec{x}$, $\vec{y}$, and $\vec{z}$ are vectors such that $A\vec{x} = 2\vec{x}$, $A\vec{y} = 3\vec{y}$, and $A\vec{z} = 5\vec{z}$, then $\det(A) = 30$.
(a) True.
(b) False.

**16.** If $A$ is diagonalizable, then $A^T$ is diagonalizable.
(a) True.
(b) False.

**17.** If $A$ is invertible and $A$ is similar to $B$, the $B$ is invertible.
(a) True.
(b) False.

**18.** If $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is a solution of a homogenous system of linear equations, then the system has infinitely many solutions.

(a) True.
(b) False.

**19.** Let $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^n$ with $\vec{a} \neq \vec{0}$. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \cdot \vec{c} = 0$, then $\vec{b} = \vec{c}$.

(a) True.
(b) False.

**20.** Let $\vec{b}, \vec{v} \in \mathbb{R}^3$. If $\vec{b} \neq \vec{0}$, then the set with vector equation $\vec{x} = \vec{b} + t\vec{v}$ is not a subspace of $\mathbb{R}^3$.

(a) True.
(b) False.