Math 636 **Final - Written Component**

1. Find a and b to obtain the best fitting equation of the form $y = a + bt^2$ for the given data.

t	-2	-1	0	1	2
y	-2	0	1	-1	-3

2. Assume that A is an $m \times n$ matrix and B is an $n \times m$ matrix where $m \neq n$. Prove that if $AB = I_m$, then $\text{rank}(B) = m$.

3. Prove Corollary 6.3.4 which states: If A is an $n \times n$ matrix with distinct eigenvalues $\lambda_1, \ldots, \lambda_k$, then A is diagonalizable if and only if $g_{\lambda_i} = a_{\lambda_i}$ for $1 \leq i \leq k$.

4. Let $\mathbb{W}$ be a subspace of a finite dimensional inner product space $\mathbb{V}$. Prove that $\text{proj}_{\mathbb{W}}(\vec{v})$ is independent of basis. That is, prove that if $\{\vec{v}_1, \ldots, \vec{v}_k\}$ and $\{\vec{w}_1, \ldots, \vec{w}_k\}$ are both orthogonal bases for $\mathbb{W}$, then

$$\frac{\langle \vec{v}, \vec{v}_1 \rangle}{\|\vec{v}_1\|^2}\vec{v}_1 + \cdots + \frac{\langle \vec{v}, \vec{v}_k \rangle}{\|\vec{v}_k\|^2}\vec{v}_k = \frac{\langle \vec{v}, \vec{w}_1 \rangle}{\|\vec{w}_1\|^2}\vec{w}_1 + \cdots + \frac{\langle \vec{v}, \vec{w}_k \rangle}{\|\vec{w}_k\|^2}\vec{w}_k$$

NOTE: Notice that our definition of $\text{proj}_{\mathbb{W}}$ is dependent on the choice of orthogonal basis. So, you cannot assume that $\text{proj}_{\mathbb{W}}(\vec{v}) = \text{proj}_{\mathbb{W}}(\vec{v})$ if we are using different bases. This is what you are supposed to prove in this question.