Math 636 **Assignment 10 - Quiz Component**

1. Consider $P_2(\mathbb{R})$ with inner product $\langle p(x), q(x)\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$.
Find the value of $\langle 1 + x - 2x^2, x + x^2\rangle$.

(a) $\langle 1 + x - 2x^2, x + x^2\rangle = 30$
(b) $\langle 1 + x - 2x^2, x + x^2\rangle = -28$
(c) $\langle 1 + x - 2x^2, x + x^2\rangle = -30$
(d) $\langle 1 + x - 2x^2, x + x^2\rangle = 31$

2. Let $A = \begin{bmatrix} 1 & 3 \\ -\sqrt{2} & 1 \end{bmatrix} \in M_{2\times 2}(\mathbb{R})$. Then

(a) $\|A\| = 9$ (b) $\|A\| = 3$ (c) $\|A\| = \sqrt{5 + \sqrt{2}}$ (d) $\|A\| = \sqrt{13}$

3. In $M_{2\times 2}(\mathbb{R})$, which of the following matrices is not orthogonal to $B = \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & -3 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -2 \\ 4 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4. Which of the following statements is false.

(a) If P is orthogonal, then P is invertible.
(b) If $\det P = \pm 1$, then P is an orthogonal matrix.
(c) If P is an orthogonal matrix, then $\det P = \pm 1$.
(d) If P is orthogonal, then $\mathrm{Col}(P) = \mathrm{Row}(P)$.

For questions 5 - 8, determine if the statement is True or False. You should make sure that you have a proof of each true statement and a counter example for each false statement.

5. The inner product of two vectors cannot be negative.

(a) True.
(b) False.

6. Let $\mathbb{W}$ be a subspace of an inner product space $\mathbb{V}$. If $\vec{u}, \vec{v} \in \mathbb{W}^\perp$, then $\vec{u} + \vec{v} \in \mathbb{W}^\perp$.

(a) True.
(b) False.

7. If $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis for an inner product space $\mathbb{V}$ and $\vec{x} \in \mathbb{V}$ such that $\langle \vec{x}, \vec{v}_i\rangle = 0$ for $1 \leq i \leq n$, then $\vec{x} = \vec{0}$.

(a) True.
(b) False.

8. If $\{\vec{v}_1, \ldots, \vec{v}_{n-1}\}$ is orthonormal in an n-dimensional inner product space $\mathbb{V}$, then there is a unique vector $\vec{v}_n \in \mathbb{V}$ such that $\{\vec{v}_1, \ldots, \vec{v}_{n-1}, \vec{v}_n\}$ is orthonormal.

(a) True.
(b) False.