Math 636 - Assignment 9 - Written Component

Due: Friday, July 15 at 4:00PM

1. Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 1 & -1 \\ -2 & 3 & 5 \end{bmatrix}$.

   (a) Find all of the eigenvalues of A and state their algebraic multiplicity.

   (b) Find a basis for the eigenspace of each eigenvalue of A.

   (c) Determine if A is diagonalizable. If it is, write a matrix P and a diagonal matrix D such that $P^{-1}AP = D$. If it isn't, explain why it isn't.

2. Let $A = \begin{bmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{bmatrix}$ where $b \neq 0$. Prove that A is diagonalizable.

3. Let $\vec{n} \in \mathbb{R}^n$, and let $L : \mathbb{R}^n \to \mathbb{R}^n$ be the linear mapping defined by

$$L(\vec{x}) = \vec{x} - 2\frac{\vec{x} \cdot \vec{n}}{\|\vec{n}\|^2}\vec{n}, \qquad \text{for all } \vec{x} \in \mathbb{R}^n$$

   (a) Show that if $\vec{y} \in \mathbb{R}^n$, such that $\vec{y} \neq \vec{0}$ and $\vec{y} \cdot \vec{n} = 0$, then $\vec{y}$ is an eigenvector of L. What is its eigenvalue?

   (b) Show that $\vec{n}$ is an eigenvector of L. What is its eigenvalue?

   (c) What are the algebraic and geometric multiplicities of all eigenvalues of L?