Math 636 - Assignment 8 - Written Component

Due: Friday, July 8 at 4:00PM

1. Find a basis for the four fundamental subspaces of $A = \begin{bmatrix} 1 & 0 & 1 & 1 & -2 \\ 3 & 1 & 4 & 1 & 1 \\ 1 & -3 & -2 & 2 & 2 \end{bmatrix}$.

2. Prove of disprove the following statement: There exists a matrix $A \in \mathbb{M}_{3\times 3}(\mathbb{R})$ such that $\texttt{Null}(A) = \texttt{Col}(A)$.

3. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear mapping defined by

$$L(x_1, x_2, x_3) = (2x_1 - x_2, x_2 + x_3, x_3 - 3x_1)$$

   and consider the basis $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ for $\mathbb{R}^3$.

   (a) Find $[L]$.

   (b) Find $[L]_{\mathcal{B}}$

   (c) Find a matrix P such that $[L]_{\mathcal{B}} = P^{-1}\,[L]\,P$

4. Let $\mathcal{B}$ and $\mathcal{C}$ both be bases for $\mathbb{R}^n$ and let $L : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator.

   (a) Prove that $[L]_{\mathcal{B}}$ and $[L]_{\mathcal{C}}$ are similar.

   (b) Prove that $\texttt{rank}\,([L]_{\mathcal{B}}) = \texttt{dim}\,(\texttt{Range}(L))$.