

1. Let $A = \begin{bmatrix} 3 & -4 & 2 \\ 1 & -2 & 2 \\ 1 & -5 & 5 \end{bmatrix}$. Which of the following is not an eigenvector of A ?

- (a) $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

2. What is the geometric multiplicity of the eigenvalue $\lambda = 2$ of $A = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 6 & 2 \\ 3 & -6 & -1 \end{bmatrix}$?

- (a) $g_\lambda = 1$
(b) $g_\lambda = 2$
(c) $g_\lambda = 3$

3. Let $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. Which of the following is not an eigenvalue of A ?

- (a) $\lambda = 0$
(b) $\lambda = 1$
(c) $\lambda = 2$
(d) $\lambda = 3$

4. Let $B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 2 & 2 \\ -5 & -1 & -4 \end{bmatrix}$. Which of the following is not an eigenvalue of B ?

- (a) $\lambda = 1$
(b) $\lambda = -1$
(c) $\lambda = 2$
(d) $\lambda = -2$

5. Which of the following matrices is diagonalizable?

(a) $\begin{bmatrix} 1 & 2 & 1 \\ -2 & 6 & 2 \\ 3 & -6 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \\ 1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 3 & 7 \\ 0 & 5 & -2 \\ 0 & 0 & -8 \end{bmatrix}$

For questions 6 - 10, determine if the statement is True or False. You should make sure that you have a proof of each true statement and a counter example for each false statement.

6. If A is diagonalizable, then A and its reduced row echelon form R have the same eigenvalues.

- (a) True.
(b) False.

7. The columns of an $n \times n$ matrix A are linearly dependent if and only if $\lambda = 0$ is an eigenvalue of A .

- (a) True.
(b) False.

8. If A and B are diagonalizable, then $A + B$ is diagonalizable.

- (a) True.
(b) False.

9. If A is invertible, then A is diagonalizable.

- (a) True.
(b) False.

10. If A is diagonalizable, then A is invertible.

- (a) True.
(b) False.