Math 636 **Assignment 8 - Quiz Component**

1. Which of the following statements is true for all 2×3 matrices A?

(a) $\dim \mathrm{Row}(A) > \dim \mathrm{Col}(A)$.
(b) $\mathrm{rank}(A) < \dim \mathrm{Null}(A)$.
(c) $\dim \mathrm{Null}(A) \geq 1$.
(d) $\dim \mathrm{Row}(A) + \dim \mathrm{Col}(A) = 2$.

2. If the reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then a basis for $\mathrm{Null}(A)$ is

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ (c) $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

3. Let $L : \mathbb{V} \to \mathbb{V}$ be a linear mapping, and let $\mathcal{B} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ be a basis for $\mathbb{V}$. If

$[L]_{\mathcal{B}} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ and $\vec{x} = 3\vec{v}_1 - \vec{v}_2 + 3\vec{v}_3$, then

(a) $L(\vec{x}) = \begin{bmatrix} 7 \\ 4 \\ 11 \end{bmatrix}$
(b) $L(\vec{x}) = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$
(c) $L(\vec{x}) = 7\vec{v}_1 + 4\vec{v}_2 + 11\vec{v}_3$
(d) $L(\vec{x}) = 5\vec{v}_1 + 4\vec{v}_2 + 11\vec{v}_3$

For questions 4 - 8, determine if the statement is True or False. You should make sure that you have a proof of each true statement and a counter example for each false statement.

4. If A is similar to B and B is similar to C, then A is similar to C.

(a) True.
(b) False.

5. If A and B are similar, then $\dim(\mathrm{Null}(A)) = \dim(\mathrm{Null}(B))$.

(a) True.
(b) False.

6. If A and B are invertible, then AB and BA are similar.
(a) True.
(b) False.

7. If the reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, then $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ is a basis for $\text{Col}(A)$.
(a) True.
(b) False.

8. If $L : \mathbb{V} \to \mathbb{V}$ is a linear mapping and $\mathcal{B}$ be a basis for $\mathbb{V}$, then $\text{Col}([L]_{\mathcal{B}}) = \text{Range}(L)$.
(a) True.
(b) False.