Math 636 **Assignment 7 - Quiz Component**

1. If $L : \mathbb{R}^2 \to \mathbb{R}^3$ is the linear mapping defined by $L(x_1, x_2) = (x_1+x_2, 2x_1+2x_2, -x_1-x_2)$, then

(a) $\text{nullity}(L) = 0$
(b) $\text{nullity}(L) = 1$
(c) $\text{nullity}(L) = 2$
(d) $\text{nullity}(L) = 3$

2. If $T : P_3(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ is the linear mapping defined by

$$T(a + bx + cx^2 + dx^3) = \begin{bmatrix} a & b+c \\ a+b & a+c \end{bmatrix}$$

then

(a) $\text{rank}(T) = 0$
(b) $\text{rank}(T) = 1$
(c) $\text{rank}(T) = 2$
(d) $\text{rank}(T) = 3$

3. Which of the following is not a linear mapping?

(a) $tr : M_{n\times n}(\mathbb{R}) \to \mathbb{R}$ (called the trace of a matrix) defined by $\text{tr}(A) = \sum_{i=1}^{n} (A)_{ii}$
(b) $M : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ defined by $M\,(a + bx + cx^2) = a + (b + c)x$
(c) $D : P_3(\mathbb{R}) \to P_3(\mathbb{R})$ defined by $D(a + bx + cx^2 + dx^3) = b + 2cx + 3dx^2$
(d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, 1)$

For questions 4 - 8, determine if the statement is True or False. You should make sure that you have a proof of each true statement and a counter example for each false statement.

4. Let $L : \mathbb{V} \to \mathbb{W}$ and $M : \mathbb{W} \to \mathbb{U}$ be linear mappings. If $\text{Range}(L) = \mathbb{W}$ and $\text{Range}(M) = \mathbb{U}$, then $\text{Range}(M \circ L) = \mathbb{U}$.

(a) True.
(b) False.

5. There exists linear mappings $L : \mathbb{V} \to \mathbb{W}$ and $M : \mathbb{W} \to \mathbb{U}$ such that $\text{Range}(M) \neq \mathbb{U}$, but $\text{Range}(M \circ L) = \mathbb{U}$.

(a) True.
(b) False.

6. If $\{\vec{v}_1, \ldots, \vec{v}_k\}$ spans $\mathbb{V}$ and $L : \mathbb{V} \to \mathbb{W}$ is linear, then $\{L(\vec{v}_1), \ldots, L(\vec{v}_k)\}$ spans $\mathbb{W}$.
(a) True.
(b) False.

7. If $L : \mathbb{V} \to \mathbb{W}$ is a linear mapping such that $\dim \mathbb{V} \geq \dim \mathbb{W}$, then $\text{Range}(L) = \mathbb{W}$.
(a) True.
(b) False.

8. If $L : \mathbb{V} \to \mathbb{W}$ and $M : \mathbb{W} \to \mathbb{U}$ are linear mapping, then $\ker(L) \subseteq \ker(M \circ L)$.
(a) True.
(b) False.