ECOS2903 - Tutorial 4 - Answers

1.(i) $\Pi(q) = p(q) \cdot q - c \cdot q - t \cdot q$
At a Stationary Point, $\Pi'(q) = p'(q)q + p(q) - c - t = 0$
Now, $\Pi''(q) = p''(q)q + 2p'(q)$. As total revenue is concave, $\Pi''(q) \leq 0$ and stationary point is a maximum.

(ii) $q'(c+t) = \frac{1}{p''(q)q+2p'(q)} \leq 0$ because the denominator is $\Pi''(q)$.

(iii) At a stationary point $T'(t) = q(c+t) + t \cdot q'(c+t) = 0$.
Now, $T''(t) = 2q'(\cdot) + tq''(\cdot) \leq 0$ because $q'() < 0$ and $q''() < 0$. Therefore SP is a maximum.

(iv) $t'(c) = -\frac{q'()+t(c)q''()}{2q'()+t(c)q''()} \leq 0$. Note denominator is $T''(t)$. An increase in c decreases q and so the marginal effect of t on T is smaller. As a result the optimal t decreases.

2. (i) $AC(q) = \frac{C(q)}{q} = \frac{F}{q} + \frac{v(q)}{q}$.

(ii) $AC'(q) = \frac{1}{q} \cdot (v'(q) - AC(q)) = 0$.

(iii) $v'(q) = AC(q)$ or marginal cost equals average cost.
(iv) $q^{*'}(F) = \frac{1}{q^*(F)} \frac{1}{AC''(q*)} > 0$.

(v) $AC^*(F) = \frac{F+v(q^*(F))}{q^*(F)}$. $\frac{dAC^*(F)}{dF} = \frac{1}{q^*(F)} > 0$

(vi) If F increases by ΔF, minimised average cost increases by approximately $\frac{\Delta F}{q^*}$, that is, the increase in average fixed costs.

3. (i) $p'(q) < 0$ so strictly decreasing and one-to-one.

(ii) Inverse exists because one-to-one. Domain of $q(p)$ is $[0, p(0)]$. $q'(p) = \frac{1}{p'(q)} < 0$

(iii) $CS(p) = \int_p^{p(0)} \; q(t)dt$

(iv) $\Pi(p) = CS(p) + (p-c) \cdot q(p)$

(v) For a stationary point, $\Pi'(p) = -q(p) + q(p) + (p-c) \cdot q'(p) = 0$. Or $p = c$.
Now $\Pi''(p) = q'(p) + p \cdot q''(p)$, for a maximum this must be ≤ 0. Assume it is.

(vi) $F = CS(c) = \int_c^{p(0)} \; q(t)dt$

(vii) The first order condition above states that maximise profit where price equals marginal cost and extract all consumer surplus with the fixed fee. (Two part tariff). You learnt this in first year.

Other Questions

4. (i) Stationary points $x = 1$ and $x = -1$. Now $f''(x) = 6x$. $f''(1) = 6 > 0$ so function is convex around $x = 1$ and it is a local minimum. $f''(-1) = -6 < 0$ so function is concave around $x = -1$ and it is a local maximum.
(ii) Stationary points $x = 0$ and $x = -3$. Now $f''(x) = e^x(6x + 6x^2 + x^3)$. $f''(0) = 0$ so $x = 0$ is an inflection pt. $f''(-3) > 0$ so function is convex around $x = -3$ and it is a local minimum.

5. Stationary points at $x = 1$ and $x = -1$. $f''(x) = \frac{2x(x^2-3)}{(1=x^2)^3}$. Inflection points at $x = 0$, $x = \sqrt{3}$, $x = -\sqrt{3}$. At these three points the second derivative changes sign.

6. (a) $L^* = 160$ and $L^{**} = 120$. Check that maximum.

(b) No. Marginal product of labour goes through the maximum of average product. You learnt this last year.

7. (a) $\int \frac{1}{\sqrt{x}} dx = 2x^{\frac{1}{2}} + C$

(b) $\int 3e^{-2x} dx = -\frac{3}{2}e^{-2x} + C$

8. (a) 8

(b) $e - \frac{1}{e}$

9. $F'(x) = x^2 + 2$ and $G'(x) = (x^4 + 2) \cdot 2x$.