# ECOS2903 - Tutorial 4

**1.** A monopolist has an indirect demand function given by $p(q)$, where $p'(q) < 0$. The monopolist's marginal cost is constant and equal to $c$. In addition, the monopolist has to pay a per-unit tax of $t$ on output it produces. The monopolist is a profit maximizers and chooses $q$.

(i) Write down an expression for profit and the necessary condition for an interior stationary point. Assume total revenue is concave. Check that the stationary point is a maximum.

(ii) The condition in (i) implicitly defines $q(c+t)$. Find an expression for $q'(c+t)$ and sign it.

(iii) The government sets the tax to maximize tax revenue, $T = t \cdot q(c+t)$. Write down the necessary condition for an interior stationary point. Assume $q''(c+t) < 0$. Check that the stationary point is a maximum.

(iv) The condition in (iii) implicitly defines $t(c)$. Find an expression for $t'(c)$ and sign it. Provide some intuition for this result.

**2.** Total cost is given by $C(q) = F + v(q)$, where $F$ is fixed cost and $v(q)$ is variable cost with $v'(q) > 0$ and $v''(q) > 0$.

(i) Write down an expression for average cost, $AC(q)$.

(ii) Write down the first-order condition for the $q$ that minimises average cost. Assume the second-order condition for a minimum is satisfied.

(iii) Write this first order condition in terms of average and marginal cost.

(iv) This first-order condition implicitly defines $q^*(F)$. Find an expression for ${q^*}'(F)$ and sign it.

(v) Substitute $q^*(F)$ into $AC(q)$ to get $AC^*(F)$. Differentiate $AC^*(F)$ with respect to $F$ and simplify using (ii).

(vi) Sign the derivative in (v) and interpret.

**3.** Let the indirect demand function be given by $p(q),\ q \in [0, \bar{q}]$, where $p(\bar{q}) = 0$ and $p'(q) < 0$.

(i) Graph $p(q)$ on a diagram with $p$ on the vertical axis. Is $p(q)$ one-to-one?

(ii) The direct demand function is given by $q(p)$ and is the inverse of the indirect demand function. What is its domain and what is the sign of $q'(p)$?

(iii) A monopolist can charge a fixed fee and a per-unit price. The fixed fee equals consumer surplus at price $p$. Write an expression for consumer surplus as a function of $p$. (hint: use the direct demand function)

(iv) Marginal cost is constant and equal to $c$. The monopolist's total profit is the sum of the fixed fee and the profit it earns by selling output at price $p$. Write down an expression for the monopolist's total profit as a function of $p$.

(v) Assume an interior solution. Find the price that maximises the monopolist's profit. (check that it is a maximum)

(vi) Write an expression for the fixed fee that maximises profit.

(vii) Discuss the economics.

**Other Questions**

**4.** Determine possible local extreme points for

(i) $f(x) = x^3 - 3x + 8$

(ii) $f(x) = x^3 \cdot e^x$

**5.** Decide whether $f(x) = \frac{x}{1+x^2}$ is convex and determine possible inflection points.

**6.** A firm's production function is $Q(L) = 12L^2 - \frac{1}{20}L^3$, where denotes the number of workers, with $L \in [0, 200]$.

(a) What size of the work force, $L^*$, maximises output? What size of the workforce, $L^{**}$, maximises output per worker?

(b) Note that $Q'(L^{**}) = \frac{Q(L^{**})}{L^{**}}$. Is this a coincidence?

**7.** Find the following integrals.

(a) $\int \frac{1}{\sqrt{x}} dx$

(b) $\int 3e^{-2x} dx$

**8.** Compute the area bounded by the graph of the function over the indicated interval.

(a) $f(x) = 3x^2$ over $[0, 2]$.

(b) $f(x) = \frac{1}{2}(e^x + e^{-x})$ over $[-1, 1]$.

**9.** Put $F(x) = \int_0^x (t^2 + 2) dt$ and $G(x) = \int_0^{x^2} (t^2 + 2) dt$. Find $F'(x)$ and $G'(x)$.