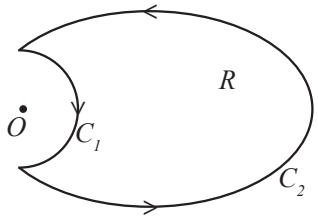
The University of Sydney
School of Mathematics and Statistics

Vector Calculus Assignment

MATH2061/2067: Vector Calculus — Semester 1, 2016

This assignment is due by Friday 20 May 2016 at 4:00pm and is worth 5% of your assessment for Vector Calculus. Submit assignments using turnitin.

Check that EVERY page of your assignment is legible and the correct way up before hitting the CONFIRM button



Let R be the region shown above bounded by the curve $C = C_1 \cup C_2$.

C_1 is a semicircle with centre at the origin O and radius $\frac{9}{5}$.

C_2 is part of an ellipse with centre at $(4, 0)$, horizontal semi-axis $a = 5$ and vertical semi-axis $b = 3$.

1. (a) Parametrise C_1 and C_2. *Hint: Use $t : -t_0 \to t_0$ as limits when parametrising C_2 and explain why* $\cos(t_0) = -\frac{4}{5}$ *and* $\sin(t_0) = \frac{3}{5}$.

(b) Calculate

$$\oint_C \mathbf{v} \cdot d\mathbf{r}$$

where $\mathbf{v} = \frac{1}{2}(-y\mathbf{i} + x\mathbf{j})$.

(c) Use Green's theorem and your answer from 1(b) to determine the area of R and then verify that it is less than πab.

2. (a) Give the cartesian equation for the ellipse used to define C_2.

(b) Show that $9 + 4r\cos\theta = 5r$ is the equation of that ellipse when written in polar coordinates (r, θ). *Hint: Square both sides first.*

(c) Calculate

$$\iint_R \frac{1}{r^3} dA$$

using polar coordinates. *Hint: Integrate with respect to r first and then θ. Explain why the limits on the outer integral should be $\theta = \pm\frac{\pi}{2}$.*

3. If $T(\mathbf{r}) = T_0/r^3$ is the temperature profile in the region R, then use the previous results to calculate the average temperature in R when $T_0 = 1000$. Verify that the average temperature is between the minimum and maximum temperatures in R.