MAE 384. Advanced Mathematical Methods for Engineers.
Homework Assignment 5. Due March 15.

1. The following data represent the vibration amplitude in mm at a location on a helicopter fuselage panel during one-half rotation of the (two-bladed) rotor.

$$y = \begin{bmatrix} 3.5 & 6.0 & 9.0 & 11.7 & 14.1 & 15.3 & 8.5 & 8.0 & 9.1 & 10.1 \\ 10.9 & 8.6 & 1.05 & -1.9 & -3.2 & -4.7 & -4.8 & -3.9 & -1.7 & 0.5 \end{bmatrix}$$

   (a) Find the real discrete Fourier transform for this data set.

   (b) Any term in the Fourier series can be written:

$$a_k \cos(k\omega t) + b_k \sin(k\omega t) = c_k \cos(k\omega t + \phi_k)$$
$$\text{where } c_k = \sqrt{a_k^2 + b_k^2} \text{ and } \phi_k = \tan^{-1}\frac{-b_k}{a_k}$$

   Find the c_k's (including c_0) and plot their amplitude on a bar graph vs. k to illustrate the relative size of each term in the series. Explain the significance of the plot.

   (c) Plot the function (inverse Fourier transform) and the original data on the same plot. Would you feel comfortable using this function to interpolate the data? Why or why not?

   (d) Estimate the maximum *velocity* amplitude at this location on the fuselage panel.

2. The density of the earth varies with radius, r. The following table gives the approximate density at different radii:

r (km)	0	800	1200	1400	2000	3000	3400	3600	4000	5000	5500	6370
ρ (kg/m^3)	13000	12900	12700	12050	11650	10600	9900	5500	5300	4750	4500	3300

   The mass of the earth can be calculated by integrating the density:

$$m = \int_0^R \rho 4\pi r^2 dr$$

   where $R =$ the maximum radius of the earth $= 6370$ m.

   (a) Find the mass of the earth using the trapezoidal rule.

   (b) Find the mass of the earth by first interpolating the data using a cubic spline to obtain 50 equally spaced intervals. Then use both trapezoidal rule and Simpson's rule to find the mass of the earth.

   (c) Discuss your results, and compare with the actual mass of the earth. In order to explain your results, you may wish to plot both the data and the interpolated data.