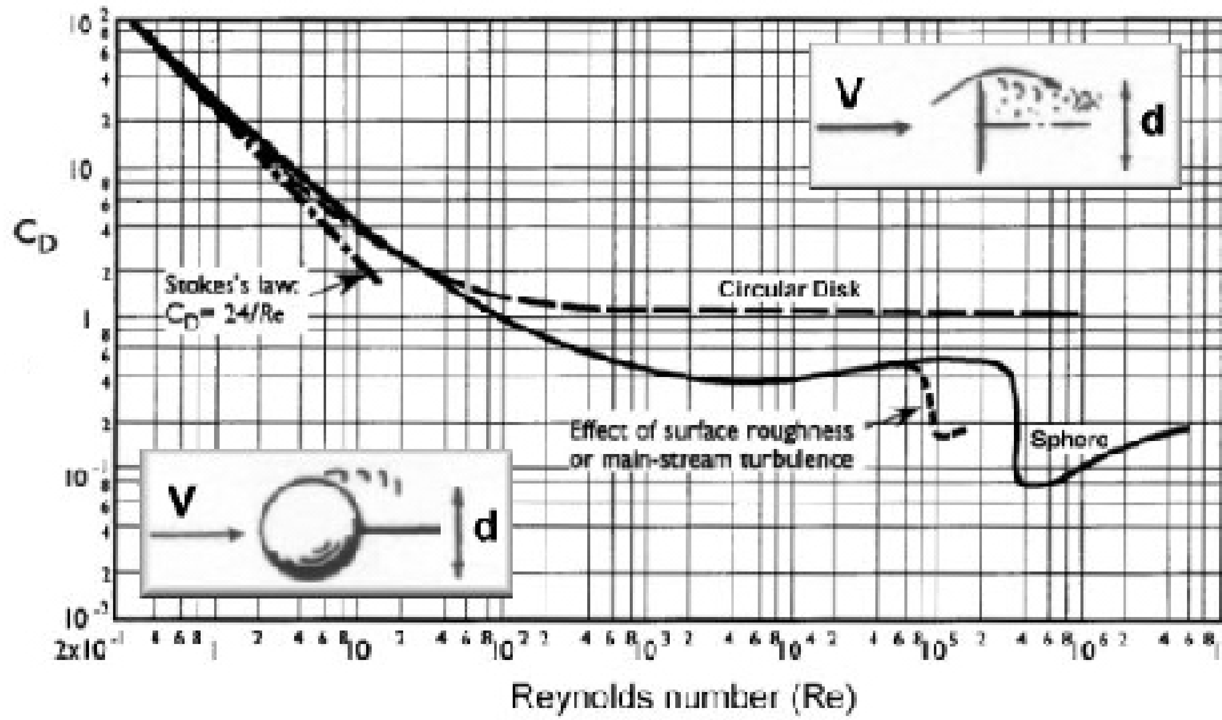
MAE 384. Advanced Mathematical Methods for Engineers.
Homework Assignment 4. Due February 22.

1. The figure shows the accepted relationship between drag coefficient and Reynolds number for a smooth sphere. Note that the curve is plotted on a log-log scale. Assume that you perform an experiment to verify this relationship. Your data for C_D[1] vs. Re[2] is shown in the table below the figure.



Reynolds number	2	20	200	2000	20,000	40,000	200,000	400,000	2,000,000
Drag coefficient	13.9	2.72	0.8	0.401	0.433	0.47	0.40	0.0775	0.214

Note: You should do the analysis in parts (a)-(e) for $\log_{10}(C_D)$ vs. $\log_{10}(\text{Re})$ rather than for C_D vs. Re. I suggest you create new variables $x = \log_{10}(\text{Re})$ and $y = \log_{10}(C_D)$ and use them in your analysis.

(a) By solving a linear system (Vandermonde matrix), find the polynomial that passes through all x-y data points. Plot the polynomial curve. On the same graph, plot the data points. Note that, ideally for all plots in this problem, you should convert your x-y data back to Re-C_D and plot on a log-log scale using the MATLAB `loglog` command. Using your polynomial fit, predict the drag coefficient at Reynolds numbers of 100, 10,000 and 1,000,000.

[1] Drag coefficient, C_D, is defined as the total drag divided by the dynamic pressure × the sphere cross-sectional area. It is a dimensionless number.

[2] The Reynolds number, Re, is defined as the ratio of inertial forces to viscous forces in a fluid flow. It is a dimensionless number.

(b) Your lab partner believes that you may have made some errors in your measurements, and the partner decides that the best fit to the x-y appears to be a quadratic. Find the quadratic either by using MATLAB's `polyfit` command, or by doing your own least-squares fit. Create a new plot showing the quadratic curve and the data points. Predict the drag coefficient at Reynolds numbers of 100, 10,000 and 1,000,000.

(c) Use the MATLAB function `spline` to interpolate the data as cubic splines. You can call the function by typing

```
>>f = spline(x,y,xi);
```

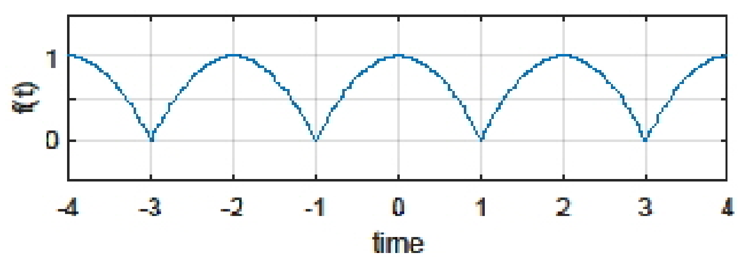
`xi` is a vector of x-locations at which the interpolation is to be completed. `f` will be a vector of interpolated (logarithms of) drag coefficients. Plot the curve and the data points. Predict the drag coefficient at Reynolds numbers of 100, 10,000 and 1,000,000.

(d) Compare the drag coefficient predictions from each of the three methods. Discuss, including other observations about the three approximations. Which do you think is likely to give the most physically realistic result?

(e) With the help of the MATLAB function `csape`, repeat the interpolation for end conditions corresponding to the natural ('variational') spline and/or the default ('Lagrange') spline and compare with results for the MATLAB default "not-a-knot" end condition. Comment on the differences. Which do you think is likely more correct and why?

2. Consider the periodic function

$$f(t) = 1 - t^2, \; -1 \le t \le 1, \, T = 2$$



(a) Find the Fourier coefficients for this function.

(b) Plot the function and its Fourier approximation between $t = -2$ and $t = 2$. For the Fourier approximation, use $K = 5, 11$ and 21. Comment.

(c) How many terms in the Fourier series are required for good resolution of the function?