ECO 303 Problem Set 3

Deadline: Friday, April 28, AT THE BEGINNING OF CLASS

[I will collect all works and then discuss the solutions in class, so you may wish to make a copy of your work before submitting]

1. A closed economy has the following Cobb-Douglas production function: $F(K, L) = K^{1/6}(EL)^{5/6}$. The depreciation rate is 1% and the saving rate is 48%. The economy is in a steady state, where the population decreases at a rate 1%, while real GDP per capita grows at a rate 1.5%.

(a) Find the growth rates of the following variables:

(i) the effective labor force, EL

(ii) the ratio of labor to capital, L/K

(iii) the labor income, wL

(iv) the ratio of the real rental rate to the real wage, r/w

(b) If total capital K is 64 million this year, find real GDP next year.

(c) By how many percentage points should the government change the saving rate so that the economy may converge to the golden rule steady state (use a "+" for an increase and a "−" for a decrease)? How would the current generation feel about the change?

2. A closed economy has two factors of production: capital and labor. The production function is known to exhibit constant returns to scale. The capital stock is about 3 times one year's real GDP. Approximately 10% of GDP is used to replace depreciating capital. Labor income is 85% of real GDP. Real GDP grows at an average rate of 3% per year. Assume the economy is at a steady state.

(a) Is the capital per effective worker lower or larger than it would have been in the golden rule steady state? [To receive points on this question, you need to show me your calculations]

(b) [growth accounting] If the population grows at a rate 1% per year, find what portion of output growth is due to:

(i) an increase in capital;

(ii) an increase in labor;

(iii) an increase in total factor productivity.

3. A closed economy has a production function: $Y = K^{1/3}L^{2/3}$, where K denotes machines and L denotes workers. The population grows at a rate 2% per year and there is no technological progress. The depreciation rate is 3%. The saving rate, s, depends on the level of capital per worker, k, as follows:

$$s = \begin{cases} 5\% \text{ if } k \leq 5 \\ (7k - 30)\% \text{ if } 5 < k \leq 10 \\ 40\% \text{ if } k > 10 \end{cases}$$

There are three steady states with $k > 0$: a low-income stable steady state, a middle unstable steady state, an a high-income stable steady state. Capital per worker in the middle unstable steady state is 6.8661.

(a) Find output per worker in the low-income stable steady state

(b) Find output per worker in the high-income stable steady state

(c) Assume the economy is in the low-income stable steady state. A donor is discussing a development aid in the form of a large-scale capital investment. What is the minimum integer amount of machines per worker that the donor should provide so that the economy could escape the poverty trap and eventually converge to the high-income stable steady state? [In answering the question, assume there are no leakages, transaction costs, or any other frictions.]

4. Consider a closed economy and use graphical analysis to illustrate how the equilibrium output, price level, and interest rate would be affected in the short run and over time by:

(a) a substantial increase in credit card usage (absent any policy response)

(b) a stock market crash (absent any policy response)

(i) What can the government do to stabilize output?

(c) an exogenous increase in the price of oil (absent any policy response)

(i) What can the Fed do to stabilize the interest rate?

5. Consider a closed economy where:

$$C = 50 + 0.8(Y - T)$$
$$G = 50,\ T = 100,$$
$$I = 150 - 10r, \text{ where } r \text{ is measured in percent}$$
$$M/P = Y - 10r \text{ , where } r \text{ is measured in percent}$$
$$M = 1,000,\ P = 2$$

(a) Assume that government spending G decreases by 10% and tax revenue T decreases by 4%

(i) Calculate the corresponding horizontal shift in the IS curve.

(ii) Calculate the resulting change in the equilibrium income and the resulting change in the equilibrium interest rate.

(iii) How would the price level evolve over time (increase, decrease, or remain the same)?

(b) Assume that government spending and tax revenue are as before: $G = 50$ and $T = 100$, but the Fed increases money supply M by 10%.

(i) Calculate the vertical shift in the LM curve.

(ii) Find the short-run equilibrium income and interest rate.

(iii) How would the price evolve over time (increase, decrease, or remain the same)?