GNG1106 – Fundamentals of Engineering Computation
Course Project
Civil Engineering
Water Pressure Exerted on a Dam

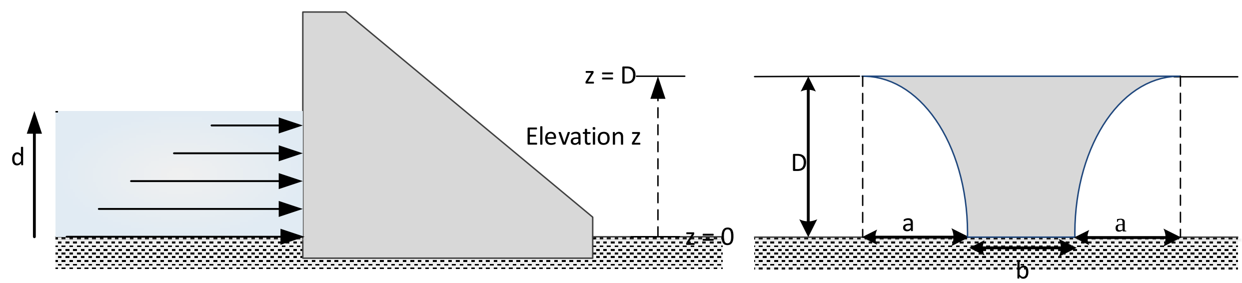
Water in a river channel exerts pressure on the upstream face of a dam as shown in Figure 1. The pressure can be characterized by

$$p(z) = \rho g(d - z) \qquad \text{(Equation 1)}$$

where

$p(z)$ = pressure in pascals (or N/m^2) exerted at an elevation z meters above the channel bottom;
g=acceleration due to gravity (9.8 m/s^2);
ρ = density of water which can be assumed to be a constant 103 kg/m^3;
d = elevation (in m) of the water above the channel bottom

According to equation 1, pressure increases linearly with depth, as depicted in Figure 1 (a).



(a) (b)

Figure 1 – Pressure of Water Exerted on a Dam. (a) Side view showing that pressure increases with depth (b) Front view showing how width of canal varies with depth.

Omitting atmospheric pressure (because it works against both sides of the dam face and essentially cancels out), the total force f_t can be determined by multiplying pressure with the area of the canal (as shown in Figure 1 (b)). Because both pressure and area vary with elevation, the total force for the depth d is obtained by evaluating

$$\begin{aligned} f_t(d) &= \int_0^d p(z)w(z)dz \\ &= \rho g \int_0^d w(z)(d-z)dz \end{aligned} \qquad \text{(Equation 2)}$$

where $w(z)$=width of the dam face (m) at elevation z (Figure 1 (b)). The channel bank is modelled using a 2nd order polynomial, such that the $w(z)$ varies as follows:

$$w(z) = \frac{2a}{D^2} z^2 + b \qquad \text{(Equation 3)}$$

where

a is the distance from the edge of the channel base to the bank of the channel,
b is the width of the channel base;
D is the depth of the channel.

Substituting Equation 3 into Equation 2 and integrating provides the following analytical solution (see Annex A).

$$f_t(d) = \rho g \left[\frac{ad^4}{6D^2} + \frac{bd^2}{2} \right] \qquad \text{Equation 4}$$

Although an analytical solution is possible for some applications, it may become challenging or even impossible to find one for many problems. Numerical methods can be used to provide solutions directly from the definite integral. A simple numerical method for integration is the Trapezoidal rule.

Recall that the Trapezoidal rule applied to the definite integral $\int_{x_0}^{x_n} f(x)dx$ is:

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[f(x_0) + \left[2\sum_{i=1}^{n-1} f(x_i) \right] + f(x_n) \right] \qquad \text{Equation 5}$$

where $h = (x_n - x_0)/n$ is the step size which divides the range of x into n equal segments which varies x from x_0, x_1, x_2, … to x_n. Applying the result of Equation 5 to Equation 2 provides the following numerical equation for computing the total force f_t on the dam as a function of water depth d.

$$f_t(d) = \rho g \frac{h}{2}\left[w(z_0)(d - z_0) + \left(2\sum_{i=1}^{n-1} w(z_i)(d - z_i) \right) + w(z_n)(d - z_n) \right] \qquad \text{Equation 6}$$

Given that $z_0 = 0$, $z_n = d$, $w(z_0) = b$, and $w(z_n) = b + 2a$, Equation 6 becomes:

$$f_t(d) = \rho g \frac{h}{2}\left[bd + \left(2\sum_{i=1}^{n-1} w(z_i)(d - z_i) \right) \right] \qquad \text{Equation 7}$$

where

$z_i = z_{i-1} + h$ for i = 1, 2, … n-1;

$w(z_i) = \frac{2a}{D^2} z_i^2 + b$;

$h = d/n$.

Develop software that allows the user to study how the total force with vary with water depth for the type of dam presented. The user will provide dam and channel dimensions (values of a, b, D), the range of water depth d as well as an increment value for plotting the total force f_t as a function of the depth d. Use the analytical solution for developing test cases. In your design, you will need to define a method to create a step size for applying the trapezoidal rule.

When new input values are given, the user is given the option to save them into a file; up to five sets of values can be stored. Thus when the software starts the user can elect to use one of the five stored values or enter new values.

Annex A

Analytical solution for *f(d)*

$$
\begin{aligned}
f_t(d) &= \rho g \int_0^d w(z)(d-z)dz \\
&= \rho g \int_0^d \left(\frac{2a}{D^2} z^2 + b \right)(d-z)dz \\
&= \rho g \int_0^d \left[\frac{2ad}{D^2} z^2 - \frac{2a}{D^2} z^3 + bd - bz \right] dz \\
&= \rho g \int_0^d \left[-\frac{2a}{D^2} z^3 + \frac{2ad}{D^2} z^2 - bz + bd \right] dz \\
&= \rho g \left[-\frac{a}{2D^2} z^4 + \frac{2ad}{3D^2} z^3 - \frac{b}{2} z^2 + bdz \right] \Bigg|_0^d \\
&= \rho g \left[-\frac{ad^4}{2D^2} + \frac{2ad^4}{3D^2} - \frac{bd^2}{2} + bd^2 \right] \\
&= \rho g \left[\left(\frac{2}{3} - \frac{1}{2} \right) \frac{ad^4}{D^2} + \frac{bd^2}{2} \right] \\
&= \rho g \left[\frac{ad^4}{6D^2} + \frac{bd^2}{2} \right]
\end{aligned}
$$