

**References:** Chap. 10, sections intro., 10.2, 5, 6

**Physics Ideas:** Vector Rotational Mechanics  
Angular Momentum & Torque

**Math Skills:** Geometry, Trigonometry, Algebra, Calculus  
Vector Geometry, Algebra, Components, & Cross Product

**Learning Goals:** (Be sure you understand where and how each goal in each assignment applies to our homework, discussion, lecture, and lab activities.)

- \* Define angular momentum, and calculate it for objects moving through space and / or rotating.
- \* State the basic relationship between angular momentum and torque, and use it to determine how angular momentum of a system changes.
- \* Tell when angular momentum of a system is conserved, and use the principle of *Conservation of Angular Momentum* to calculate and relate changes in the motion of an object or system of objects.
- \* State the basic relationship between angular momentum and torque, and use it to determine how angular momentum of a system changes.
- \* Tell when angular momentum of a system is conserved, and apply the principle of *Conservation of Angular Momentum* when appropriate.
- \* Show that algebraic & numerical results have correct units and are physically reasonable.

**For extra practice:** Chapter 10: Qs #Q10.20, 22, 24-27; Es & Ps #10.43, 47, 51, 89, 95, 97, 99 (not turned in)

**#10.42 [Block on Table] Please add: (e)** Show that the *linear speed* of the block has also increased. What supplies energy to the block to increase its speed and kinetic energy?

**#10.46 [Mud on Door] Please add: (b)** How would the mud's *moment of inertia* be included?

**To be turned in to your Discussion instructor's HW box by 4 PM on Friday, April 10:**

**#10.41 [Neutron Star] Please add: (b)** What is the rotation *period* of the neutron star? How many times does it rotate in one second? **(c)** Show that the star's *rotational kinetic energy* has increased, and determine by what factor. What provides the added kinetic energy?

[HINT: Show that you can write the star's KE as  $K = L^2 / 2I$ , where  $L$  and  $I$  are angular momentum and moment of inertia.]

**#10.45 [Parachutist on Turntable] Please add: (c)** Is *horizontal linear momentum* of the parachutist + turntable conserved here? Why or why not?

**[Assignment CONTINUES on next page]**

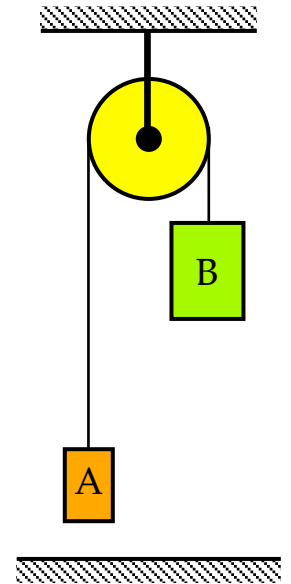
#1. **[Atwood Machine]** Two blocks A and B with masses  $m_A$  and  $m_B (> m_A)$  hang from a light string passing over a uniform solid cylindrical wheel without slipping. The wheel's mass is  $M$ , its radius is  $R$ , and friction due to its axle is negligible. The blocks are released from rest at the locations shown. In a previous HW problem, we analyzed this system using *Conservation of Energy* and *Newton's 2nd Law for Rotation*. Here, we'll analyze this problem by *angular momentum* considerations

(a) Draw a *free-body diagram* showing all the *external forces* acting on the system of 2 blocks + wheel + string. Be sure that you show only external forces for this system, not internal forces acting within the system.

(b) Write an expression for the *net external torque* acting on the system of the blocks + wheel + string about the wheel's axle. What is its vector direction?

(c) Write an expression for the magnitude of the system's *total angular momentum* about the wheel's axle when each block is moving with speed  $v$  and the wheel has corresponding angular speed  $\omega$ . What is the vector *direction* of this angular momentum? What is the kinematic relationship between  $v$  and  $\omega$ ?

(d) Write the relation between the *net external torque* and *total angular momentum* from the previous parts, and use it to obtain an expression for the *acceleration* of each block. Check your results with Problem #1 on HW #9.

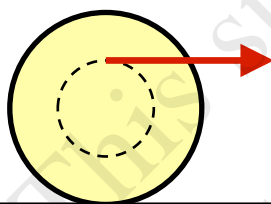


#2. **[Rolling Spool]** A spool on a horizontal surface with friction can be pulled by a string wrapped around the middle of its axle. For each case below (a-c), in which *direction* will the spool roll when the string is pulled gently? Be sure your reasoning is clear. [HINT: Consider the

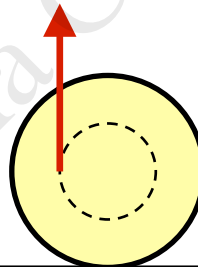
relation  $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$  about the point on the ground midway between the

contact points of the spool's wheels with the surface. Refer to your lecture notes to recall how the spool's translational and rotational angular momentum about that point are related.]

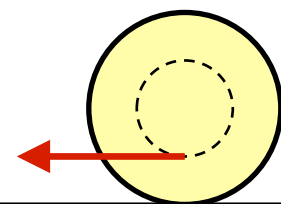
(a)



(b)



(c)



(d) Show that there is a *direction* in which the string can be pulled gently so the spool slides along the surface without rotating. Illustrate this direction on a *diagram*, and show how to determine an expression for the angle the string makes with the vertical. Be sure your reasoning is clear.

[Assignment CONTINUES on next page]

**#3. [Colliding Sticking Disks]** Two identical uniform solid disks, each of mass  $M$  and radius  $R$ , can slide on a horizontal frictionless surface. Disk A is initially moving with speed  $v_0$  without spinning. Disk B is initially at rest without spinning. Disk A approaches disk B and makes grazing collision with B so their edges just barely touch and then stick together. The disks then proceed to slide stuck together while spinning about their contact point.

(a) Which of these physical quantities do you know are *conserved* in this collision, and which are not: linear momentum, kinetic energy, angular momentum? Why?

(b) What is the final *speed* of the center-of-mass (CM) of the stuck-together disks?

(c) What is the *moment-of-inertia* of the stuck-together disks about their CM? [HINT: You'll need to use the *Parallel-Axis Theorem* for each disk + superposition.]

(d) What is the *angular speed* of rotation of the stuck-together disks? [HINT: You can consider the motion as seen in the CM reference frame—the reference frame in which the CM is at rest.]

(e) What *fraction* of the initial *kinetic energy* (KE) remains as *total KE* after the collision?

(f) In order for the two stuck-together disks not to rotate after they collide, in which *sense* (CW  $\curvearrowright$  or CCW  $\curvearrowleft$ ) should the incoming disk be rotating? What should its initial *angular speed* be?

