

**CME – Controle de Máquinas Elétricas – 2016**

Due Date: Thursday, October 27

Name \_\_\_\_\_

**Required Homework #2**

**Note:** Please fill in the answers on these sheets to simplify grading but attach separate sheets that include all calculations used to complete your homework so that the grader can assign partial credit when appropriate.

1. A 100 hp squirrel-cage induction machine has the following parameters and characteristics:

460 V (line to line); 60 Hz; 4 poles;

$Z_B = 2.84 \Omega$ ;  $V_B = 376 \text{ V}$ ;  $I_B = 132 \text{ A}$ ;  $T_B = 395.8 \text{ Nm}$  (base values)

$r_1 = r_2 = 0.015 \text{ pu} = 0.0425 \Omega$

$x_1 = x_2 = 0.100 \text{ pu} = 0.284 \Omega$

$x_m = 3.0 \text{ pu} = 8.51 \Omega$

Rated Operation:  $s_R = 0.0177$ ;  $I_R = 1.17 \text{ pu}$ ;  $\cos \theta_R = 0.884$

Assume that the machine is initially operating at rated load with rated voltage and frequency. A wye-connected resistive load drawing 0.25pu power (on the same 100 hp base) is connected in parallel with the stator terminals of the machine. At time  $t = 0$ , when the complex excitation voltage vector is aligned with stator phase  $a$ , the three-phase voltage source is simultaneously disconnected from all three phases of the parallel-connected machine and resistive load. Assume that the rotor inertia is infinite so that the rotor speed is fixed at its initial value.

a) Find the constant-speed eigenvalues in the *stator* reference frame that characterize the switching transient following removal of the source.

$$\underline{\lambda}_1 = \text{_____} \text{ rad/s}$$

$$\underline{\lambda}_2 = \text{_____} \text{ rad/s}$$

b) Find the initial values of the complex vectors representing the stator and the rotor flux linkages in the stator reference frame.

$$\underline{\lambda}_{qds0}^s = \text{_____} \text{ pu}$$

$$\underline{\lambda}_{qdr0}^s = \text{_____} \text{ pu}$$

c) Find the time domain expressions for the complex vectors representing the stator voltage  $\underline{v}_{qds}^s$  the stator current  $\underline{i}_{qds}^s$ , the rotor flux linkage  $\underline{\lambda}_{qdr}^s$ , and the torque  $T_e$  following the switching event. Use MATLAB or some other computer program to plot,  $v_{as}(t)$ ,  $i_{as}(t)$ ,  $|\underline{\lambda}_{qdr}^s| = \lambda_r(t)$ , and in per-unit on separate axes between  $t = 0$  and 0.1s.

$$\underline{v}_{qds}^s = \underline{\quad\quad\quad} \text{pu}$$

$$\underline{i}_{qds}^s = \underline{\quad\quad\quad} \text{pu}$$

$$\underline{\lambda}_{qdr}^s = \underline{\quad\quad\quad} \text{pu}$$

$$T_e = \underline{\quad\quad\quad} \text{pu}$$

- d) Solve the same problem again using the transient equivalent circuit, assuming constant rotor flux linkage. Determine the values of voltage behind transient reactance  $\underline{E}'_{qd}$ , the stator transient reactance  $\underline{X}'_s$ , and a time domain expression for the approximate complex stator current vector  $\underline{i}_{qds}^s$ . Plot the phase *a* current waveforms  $i_{as}(t)$  from the previous solution in part c) and the solution resulting from this transient equivalent circuit on the same figure for the first 3 cycles only. Clearly label the waveforms.

$$\underline{E}'_{qd} = \underline{\quad\quad\quad} \text{pu}$$

$$\underline{X}'_s = \underline{\quad\quad\quad} \text{pu}$$

$$\underline{i}_{qds}^s = \underline{\quad\quad\quad} \text{pu}$$

- e) Determine an approximate value for the time it takes for the stator voltage amplitude to drop to 50% of rated voltage (0.5pu), using the solution from part c).

$$t_{50} = \underline{\quad\quad\quad} \text{sec}$$

2. The 100 hp, 460 volts, 60 Hz, three-phase, 4-pole squirrel-cage induction machine of Problem 1 has the following per-unit parameters (repeated here for convenience):

$$r_s = 0.015; \quad r_r = 0.015; \quad x_{ls} = 0.1; \quad x_{lr} = 0.1; \quad x_m = 3.0; \quad M = 2 \text{ sec}$$

Assume that the machine is initially operating at the same steady-state pre-fault speed conditions as in Problem 1 ( $s = 0.0177$ ) with rated voltage excitation and the voltage excitation vector aligned with phase *a* at  $t = 0$ . As described in Problem 1, a wye-connected resistive load drawing 0.25 pu power (on the same 100 hp base) is connected in parallel with the stator terminals of the machine. The machine is driving a load that has the following torque-speed relationship:  $T_L = 1.055 \omega_r^2$  where both  $T_L$  and  $\omega_r$  are in pu and the value of the coefficient has been selected to establish steady-state operation at the rated operating point as described above. The value of the mechanical time constant  $M$  given above is the total combined value for the machine and the mechanical load.

- a) Solve the same problem described in part c) of Problem 1 (voltage excitation removal from all three phases of the paralleled resistive load and machine at time  $t = 0$ ) by simulation using MATLAB or whatever simulation software you prefer. However, you are *not* permitted to use any prepared modules or blocks for the ac machine equations; you must program them yourself. Carry out your simulations in the synchronous reference frame using flux voltages expressed in per-unit values as the state variables. (Assume that the axes of the synchronous and stationary

frames are aligned at  $t = 0$ ). Provided per-unit plots of the rotor flux voltage amplitude  $\psi_R$ , the phase-to-neutral phase  $a$  stator voltage  $v_{as}$  (in the stationary frame), the phase  $a$  stator current  $i_{as}$  (also in the stationary reference frame), and the torque  $T_e$  throughout the event. Begin the simulation at  $t = 0$  and continue it for 0.1 sec. Plot all quantities in per-unit. Assume that the rotor speed is constant (at  $s = 0.0177$ ) during the entire event.

Hint: Insist that flux voltages (and currents) must be continuous across the “boundary” at  $t = 0$ .

- b) Repeat the solution of part a) with the speed no longer assumed constant (i.e., include the mechanical differential equation), but with an initial condition of  $\omega_R = (1-s_R) = 0.9823$  pu at time  $t = 0$ . Assume that the load torque ( $T_L$ ) follows the relationship presented above. Plot the same variables as in part a) for the entire interval between  $t = 0$  and  $t = 0.1$  s plus the rotor speed  $\omega_R$  (also in per-unit) as an additional variable.
- c) Compare the simulation answers determined in part a) and part b) to each other and to the results calculated in Problem 1 using the constant-speed analytical solution (part c)). Describe the similarities and differences. Are there differences between the torque waveforms, and, if so, why?
- d) Extend the simulation in part b) by assuming that the stator voltage is reapplied simultaneously to all three stator phases after exactly 0.1 s (Note: the excitation voltage vector phase angle is  $\omega_e t = 377 \times 0.1 = 37.7$  rad at the instant of reapplication, and the flux voltages and currents must be continuous across the transition). Continue the simulation until the rotor speed reaches 98% of its rated value again ( $\omega_{r98\%} = 0.98 \times 0.9823 = 0.9627$  pu) for the first time after the voltage is reapplied. At what time  $t_{\omega_{r98\%}}$  (measured with respect to  $t = 0$  when the excitation was first removed) does the rotor reach this speed? What is the maximum amplitude (absolute value) of the stator phase  $a$  current and the torque during this voltage reapplication transient between  $t = 0.1$  s and  $t_{\omega_{r98\%}}$ ?

$$t_{\omega_{r98\%}} = \underline{\hspace{10cm}} \text{ s}$$

$$i_{as\max} = \underline{\hspace{10cm}} \text{ pu}$$

$$T_{\max} = \underline{\hspace{10cm}} \text{ pu}$$