CSCE 4115 ASSIGNMENT #4

Due Wednesday, October 26, 2016

1. Design a context-free grammar and a pushdown automaton for each of the following languages. The PDA's should be deterministic if possible. If it is not possible to write a DPDA, give an informal justification for your claim (i.e., L is a NCFL). For each CFG and PDA you write, give an informal explanation of how and why the CFG/PDA works. Also, for each CFG, give two sample derivations to further demonstrate the correctness of your grammar, and for each PDA, run the PDA in Prolog with two correct and two incorrect strings. For the sample strings, use the strings included in the problem statement.

   (a) L = {w | w $\in \{a, b, c, d\}^+$, w contains an equal number of a's and b's or an equal number of c's and d's}. Example strings include aabbccdd, dabcbcd, and dabcab.

   (b) L = {w | w $\in \{0, 1\}^*$, w is a list of unary integers separated by 1's that are *not* in ascending order}. For example, 00010010000 is in L but 01001000 is not.

   (c) L = $\{a^i b^j x c^k | (i = k \wedge x = 0) \vee (j = k \wedge x = 1),\ i, j, k \geq 0\}$. Example strings in this language include $aab0cc$, $aaab1c$, $aa0cc$, $bbb1ccc$, and $\emptyset$.

2. Consider the pushdown automaton below:

   M = $(\{q_0, q_1\}, \{0, 1\}, \{X, Z\}, \delta, q_0, Z, \emptyset)$
   $\delta(q_0, 1, Z) = \{(q_0, XZ)\}$
   $\delta(q_0, 1, X) = \{(q_0, XX)\}$
   $\delta(q_0, 0, X) = \{(q_1, X)\}$
   $\delta(q_0, \varepsilon, Z) = \{(q_0, \varepsilon)\}$
   $\delta(q_1, 1, X) = \{(q_1, \varepsilon)\}$
   $\delta(q_1, 0, Z) = \{(q_0, Z)\}$

   Give a context-free grammar which generates L(M). Describe the language L in English and/or set notation.

3. Prove that the following are not context-free languages.

   (a) L = {w | w is a string of the form $(0^*1)^*$, with all sets of 0's the same length}. The strings of this language are ε, 0101, 001001, 00010001, ...

   (b) L = $\{0^{n!} \mid n \geq 0\}$. The strings of this language are 0, 00, 000, 000000, 0^{24}, etc.

   (c) L = {w | w $\in \{0, 1\}^*$, w contains no string of the form xx^R}.

4. Consider the grammar G with productions as follows:

   S → AD | BC
   A → a
   B → b
   C → a | AS | BE
   D → b | BS | AF
   E → CC
   F → DD

   Use the CYK algorithm to test membership of $abbaba$.