CS 577 – Sections 1 and 2
Fall 2016
Homework #1

Due at beginning of class 9/21 and 9/22

Rules for Homework.

i.) Everyone must do his or her own work. Any sources beyond your class notes and textbook must be cited. In any case, there must be significant "value added" by your work.

ii.) Grading is based on correctness and clarity. In particular, computations and results need to be explained.

1.

Consider n points arranged on a circle, connected by all possible line segments. Assume they are in *general position*, which means that no point inside the circle is on more than two of the segments. Let $R(n)$ denote the number of regions (inside the circle) thus formed.

a) Find $R(n)$ for $n = 0, 1, 2, 3, 4, 5$. You should see a pattern.

b) Looks can be deceiving! Show that

$$R(n+1) \leq R(n) + O(n^3). \tag{1}$$

c) Explain, using induction and the definition of O-notation, why your recurrence implies that $R(n) = O(n^4)$.

Hints: Suppose the first n points are arranged along the upper semicircle. Put the $(n+1)$-st point near the bottom of the circle, and connect it to the i-th "old" point. Find a formula for the number of lines crossed (it should involve n and i), and thereby determine the number of new regions created when this line is added. Proving that $R(n) = O(n^4)$ will be aided if you first replace the $O(n^3)$ in (1) by an explicit function of n.

2.

Consider the following non-recursive version of DFS, which counts the vertices reachable from s in G:

```
P(G,s)
count := 0
push s onto stack
while stack nonempty
    u:= pop stack
    if u is not marked "counted" then
        mark u "counted"
        count := count + 1
        for each neighbor v of u
            push v onto stack
return count
```

Initially all nodes are unmarked. Show:

a) At the end of each iteration of the while loop (including the 0th iteration, i.e., the beginning of the first iteration), the correct count equals `count + |A|`, where `A` is the set of uncounted nodes reachable from some node on the stack.

b) `P` terminates.

c) `P` outputs the correct count upon termination.