Homework due Wednesday, Sept. 14

1. Let A and B be languages. We say that $A \leq_m B$ if there is a computable function f such that, for every string x:

$$x \in A \Rightarrow f(x) \in B$$

and

$$x \notin A \Rightarrow f(x) \notin B.$$

(This is a very important relation on languages.)

Show that $\leq_m$ is a transitive relation. That is, show that if $A \leq_m B$ and $B \leq_m C$, then $A \leq_m C$. If $A \leq_m B$ and B is recursive, is A recursive? If B is r.e., is A recursively-enumerable?

If $A \leq_m B$ and A is recursive, is B recursive? If A is r.e., is B recursively-enumerable? Justify your answers.

2. Explain why $\leq_m$ is *not* a partial order. (As part of this exercise, you might need to do some searching, to find out what a partial order is.)

3. Let A and B be languages. We say that $A \equiv_m B$ if $A \leq_m B$ and $B \leq_m A$. Show that $\equiv_m$ is an equivalence relation.

4. Let HP denote the halting problem. Consider the language:

$$A = \{1x : x \in HP\} \cup \{0x : x \notin HP\}.$$

Show $HP \leq_m A$ and $\overline{HP} \leq_m A$. Is A recursively-enumerable? Justify your answer.

5. Show that A is decidable if and only if A can be recursively enumerated *in lexicographic order*.