Case 5-3

Farm Management

The **Ploughman family** owns and operates a 640-acre farm that has been in the family for several generations. The Ploughmans always have had to work hard to make a decent living from the farm and have had to endure some occasional difficult years. Stories about earlier generations overcoming hardships due to droughts, floods, and so forth, are an important part of the family history. However, the Ploughmans enjoy their self-reliant lifestyle and gain considerable satisfaction from continuing the family tradition of successfully living off the land during an era when many family farms are being abandoned or taken over by large agricultural corporations.

John Ploughman is the current manager of the farm, while his wife Eunice runs the house and manages the farm's finances. John's father, Grandpa Ploughman, lives with them and still puts in many hours working on the farm. John and Eunice's older children, Frank, Phyllis, and Carl, also are given heavy chores before and after school.

The entire family can produce a total of 4,000 person-hours' worth of labor during the winter and spring months and 4,500 person-hours during the summer and fall. If any of these person-hours are not needed, Frank, Phyllis, and Carl will use them to work on a neighboring farm for $5/hour during the winter and spring months and $5.50/hour during the summer and fall.

The farm supports two types of livestock, dairy cows and laying hens, as well as three crops: soybeans, corn, and wheat. (All three are cash crops, but the corn also is a feed crop for the cows and the wheat also is used for chicken feed.) The crops are harvested during the late summer and fall. During the winter months, John, Eunice, and Grandpa make a decision about the mix of livestock and crops for the coming year.

Currently, the family has just completed a particularly successful harvest that has provided an investment fund of $20,000 that can be used to purchase more livestock. (Other money is available for ongoing expenses, including the next planting of crops.) The family currently has 30 cows valued at $35,000 and 2,000 hens valued at $5,000. They wish to keep all this livestock and perhaps purchase more. Each new cow would cost $1,500, and each new hen would cost $3.

Over a year's time, the value of a herd of cows will decrease by about 10 percent and the value of a flock of hens will decrease by about 25 percent due to aging.

Each cow will require two acres of land for grazing and 10 person-hours of work per month, while producing a net annual cash income of $850 for the family. The corresponding figures for each hen are no significant acreage, 0.05 person-hours per month, and an annual net cash income of $4.25. The chicken house can accommodate a maximum of 5,000 hens, and the size of the barn limits the herd to a maximum of 42 cows.

For each acre planted in each of the three crops, the next table gives the number of person-hours of work that will be required during the first and second halves of the year, as well as a rough estimate of the crop's net value (in either income or savings in purchasing feed for the livestock).

To provide much of the feed for the livestock, John wants to plant at least one acre of corn for each cow in the coming year's herd and at least 0.05 acre of wheat for each hen in the coming year's flock.

John, Eunice, and Grandpa now are discussing how much acreage should be planted in each of the crops and how many cows and hens to have for the coming year. Their objective is to maximize the family's monetary worth at the end of the coming year (the *sum* of the net income from the livestock for the coming year *plus* the net value of the crops for the coming year *plus* what remains from the investment fund *plus* the value of the livestock at the end of the coming year *plus* income from working on a neighboring farm *minus* living expenses of $40,000 for the year).

Data per Acre Planted

	Soybeans	Corn	Wheat
Winter and spring, person-hours	1.0	0.9	0.6
Summer and fall, person-hours	1.4	1.2	0.7
Net value	$70	$60	$40

a. Identify verbally the components of a linear programming model for this problem.

b. Display the model on a spreadsheet.

c. Obtain an optimal solution and generate the sensitivity report. What does the model predict regarding the family's monetary worth at the end of the coming year?

d. Find the allowable range for the net value per acre planted for each of the three crops.

The above estimates of the net value per acre planted in each of the three crops assumes good weather conditions. Adverse weather conditions would harm the crops and greatly reduce the resulting value. The scenarios particularly feared by the family are a drought, a flood, an early frost, *both* a drought and an early frost, and *both* a flood and an early frost. The estimated net values for the year under these scenarios are shown next.

	Net Value per Acre Planted		
Scenario	Soybeans	Corn	Wheat
Drought	−$10	−$15	0
Flood	15	20	$10
Early frost	50	40	30
Drought and early frost	−15	−20	−10
Flood and early frost	10	10	5

e. Find an optimal solution under each scenario after making the necessary adjustments to the linear programming model formulated in part *b*. In each case, what is the prediction regarding the family's monetary worth at the end of the year?

f. For the optimal solution obtained under each of the six scenarios (including the good weather scenario considered in parts *a-d*), calculate what the family's monetary worth would be at the end of the year if each of the other five scenarios occurs instead. In your judgment, which solution provides the best balance between yielding a large monetary worth under good weather conditions and avoiding an overly small monetary worth under adverse weather conditions?

Grandpa has researched what the weather conditions were in past years as far back as weather records have been kept and obtained the data shown on the next page. With these data, the family has decided to use the following approach to making its planting and livestock decisions. Rather than the optimistic approach of assuming that good weather conditions will prevail (as done in parts *a-d*), the *average* net value under all weather conditions will be used for each crop (weighting the net values under the various scenarios by the frequencies in the above table).

Scenario	Frequency
Good weather	40%
Drought	20
Flood	10
Early frost	15
Drought and early frost	10
Flood and early frost	5

g. Modify the linear programming model formulated in part *b* to fit this new approach.

h. Repeat part *c* for this modified model.

i. Use a shadow price obtained in part *h* to analyze whether it would be worthwhile for the family to obtain a bank loan with a 10 percent interest rate to purchase more livestock now beyond what can be obtained with the $20,000 from the investment fund.

j. For each of the three crops, use the sensitivity report obtained in part *h* to identify how much latitude for error is available in estimating the net value per acre planted for that crop without changing the optimal solution. Which two net values