1. For each of following systems, $y[n]$ denotes the output and $x[n]$ the input. For each system, determine whether the specified input-output relationship is linear and/or shift-invariant.
   (a) $y[n] = 4x[n] + 1$
   (b) $y[n] = x[n]\sin\left(\frac{2\pi}{5}n + \frac{\pi}{8}\right)$
   (c) $y[n] = \left(x[n]\right)^2$
   (d) $y[n] = \sum_{m=n-N}^{n} x[m]$
2. Consider the sequence

$$x[n] = \begin{cases} a^n & \mathrm{n} \ge n_0 \\ 0 & \mathrm{n} < n_0 \end{cases}$$

   (a) Determine the z-transform of $x[n]$.
   (b) Determine the Fourier transform of $x[n]$. Under what conditions does the Fourier transform exist?
3. Consider the first-order system, causal system,

$$y[n] = \alpha y[n-1] + x[n]$$

   (a) Find the system function, $H[z]$, for this system.
   (b) Find the impulse response of this system.
   (c) For what values of α will the system be stable?
   (d) Assume that the input is obtained by sampling with period T. Determine the value of α such that

$$h[n] < e^{-1} \quad \text{for} \quad nT < 4\, m\text{sec}$$

   i.e., find the value of α that gives a time constant of 4msec.
4. A speech signal is sampled at a rate of 10000 samples/sec(10 kHz). A segment of length 1024 samples is selected and the 1024-point DFT is computed.
   (a) What is the time duration of segment of speech?
   (b) What is the frequency resolution (spacing in Hz) between the DFT values?
   (c) How do your answers to parts (a) and (b) change if we compute the 1024-point DFT of 512 samples of the speech signal. (The 512 samples would be augmented with 512 zero samples before the transform was computed.)