CSC463S **Problem Set 4** **Winter, 2016**

Due: Friday, April 1, beginning of tutorial

NOTE: Each problem set counts 15% of your mark, and it is important to do your own work. You may consult with others concerning the general approach for solving problems on assignments, but you must write up all solutions entirely on your own. Copying assignments is a serious academic offense and will be dealt with accordingly.

1. The decision problem PARTITION is defined on page 13 of the Notes "NP and NP-Completeness". (You may assume that $a_1, \ldots a_m$ are positive integers.)

   Define the associated search problem PARTITION-SEARCH and give an algorithm showing that
   PARTITION-SEARCH $\stackrel{p}{\rightarrow}$ PARTITION.

   Give a loop invariant for your algorithm.

   (See Definition 6 in the Notes "Search and Optimization Problems" for the definition of $\stackrel{p}{\rightarrow}$.)

2. Consider the problem DISTANCE-PATH.

   DISTANCE-PATH
   Instance
   $\langle G, s, t, d \rangle$, where G is an undirected graph, s and t are nodes in G, and d is a positive integer.

   Question Is the distance from s to t exactly d? In other words, is it the case that there is a path of length d from s to t, and no shorter path from s to t?

   (a) Show that DISTANCE-PATH $\in$ **NL**.

   (b) Show that DISTANCE-PATH is **NL**-complete.

   Hint: Show that $PATH \leq_L$ DISTANCE-PATH. Given a directed graph G construct an undirected graph G' by making n copies of G. Each edge in G' goes from copy i to copy $i+1$.

3. Use a padding argument to show that NL = coNL implies NSPACE(n^3) = coNSPACE(n^3).

   See Problem 9.13, in the textbook for a description of padding.

4. Show that $TQBF \notin$ DSPACE($n^{1/5}$). You may refer to the proof of Theorem 8.9 in the text, and assume the fact that the reduction presented there can be carried out in log space.