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Market Risk and Model Risk For a Financial Institution Writing Options

by

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ABSTRACT

Trading in derivatives involves heavy use of quantitative models for valuation and risk management. These models are necessarily imperfect, and when options are involved, the models require a volatility input that must be forecasted, subject to error. This creates "model risk" to which nearly all participants in derivatives markets are exposed. In this paper, we conduct an empirical simulation, with and without hedging, using historical data from 1976-1996 for several important markets. The object is to develop a quantitative assessment of the extent to which the different sources of model risk can be expected to affect the kind of basic option writing strategy that might be followed by a bank or another financial institution. Specifically, we explore the following problem: If a bank or a similar financial institution writes standard European calls and puts and prices them using the appropriate variant of the Black-Scholes model with a volatility forecast computed optimally from historical data, what are the risk and return characteristics of the trade? More generally, what is the market and model risk exposure faced by a bank that does this transaction repeatedly over time? The results indicate that pricing and hedging errors due to imperfect models and inaccurate volatility forecasts create sizable risk exposure for option writers. We then consider to what extent the bank can limit the damage due to model risk by pricing options using a higher volatility than its best estimate from historical data.

JEL Classification: G13, G21, G22

Introduction

With the remarkable growth of trading in derivative instruments in recent years, derivatives activities of banks and other financial institutions are accelerating. The volume of outstanding contracts expands at a rapid pace, and the contracts themselves become more complex, span longer maturities and cover a broader range of underlying assets.

Much of the growth is in “plain vanilla” instruments like forwards, for which the principles of valuation are well-established. However, assessing and managing risk, even for simple products, can entail considerable uncertainty. Derivatives with option features present significantly greater challenges with regard to correct valuation and the design and implementation of risk management strategies. Moreover, options involve the asymmetry between buying and writing, in that only the option buyer has liability limited to the amount invested, while the option writer is exposed to the risk of losses that can greatly exceed the initial premium received. Not surprisingly, the public prefers to buy options rather than to write them. Since each contract requires a buyer and a writer, if the public wants to be long options, the dealer community must be short options overall. This means that the typical financial institution entering the derivatives business in order to satisfy its customers’ demand for options, and (ideally) to earn profits by making markets, will be primarily writing contracts. In so doing, it will be exposed to a variety of risks.

Derivatives risks have been widely discussed. Often they are classified into several categories, that may differ from one discussion to another, but a common taxonomy is the following[1]:

> Market risk: the risk that movements in financial market prices will impair a firm’s financial condition due to its positions in derivatives.
>
> Credit risk: the risk (broadly defined) that the counterparty to a derivatives contract will fail to fulfill its contracted obligations.
>
> Operational risk: the risk of derivatives-related losses from deficient internal controls or information systems.
>
> Legal risk: the risk that derivatives contracts will not be legally enforceable.

Recommended procedures for managing these different risks safely have been broadly

[1] See, for example, the Group of Thirty [1993] or U.S. Government Accounting Office [1994].

promulgated.[2] Further, the growth of derivatives activity has also increased awareness of risk exposures generally, and contributed to the development of more formal methods of risk assessment, such as Value-at-Risk (VaR). Indeed, the Bank for International Settlements (BIS) has suggested regulatory policies for setting capital requirements for banks that are closely related to the VaR methodology, and the system has been adopted by the European Community (EC) and by banking authorities elsewhere around the world.[3]

One important feature of both the growing derivatives trade and also the new approaches to risk management is that the derivatives business today depends very heavily on theoretical models for pricing contracts, and for risk assessment and hedging. This introduces an important new type of risk that has not played much of a role in investing before: model risk. Since the derivatives trade is based so much on pricing models, model errors create risk: misvalued contracts may be sold for less than they are actually worth or purchased at too high prices, incorrectly estimated risk exposures may be greater than anticipated and hedging strategies may be less effective than they are supposed to be.

Figlewski [1998] discusses several sources of model risk facing derivatives traders. First, there is the risk that a given model may be misspecified. A common problem is that to derive a valuation model, it is necessary to assume a stochastic process for the derivative's underlying asset. The original Black-Scholes [1973] (BS) option pricing model assumes that the underlying follows a lognormal diffusion process. This process has a number of virtues, including the fact that (instantaneous) rates of return have a normal (Gaussian) distribution, for which the mathematics are well known, and the time independence of the stochastic component is consistent with the principle of informationally efficient markets. Many subsequent derivatives models have generalized the returns process but continued to assume the stochastic component remains locally Gaussian. However, empirical investigation almost invariably finds that actual returns are too "fat-tailed" to be lognormal. There are more realizations in the extreme tails (and the extreme values themselves are more extreme) than a lognormal distribution allows for. In other words, the standard valuation models are based upon assumptions about the returns process that are not empirically supported for actual financial markets. Other difficulties with the distributional assumptions, notably parameter instability, create additional errors in the standard models.

A second important source of risk in using a valuation model is that not all of the input parameters are observable. In particular, even if one has a correctly specified model, using it requires knowledge of the volatility of the underlying asset over the entire lifetime of the contract. This creates a formidable forecasting problem, for which neither the "best" estimation procedure nor the model risk characteristics of the resulting theoretical option

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[2] See The Risk Standards Working Group [1996], or The Basle Committee on Banking Supervision [1994].

[3] The Basle Committee on Bank Supervision [1996].

values are known.

Third, one of the important features of the option pricing paradigm is that the theoretically correct risk management strategy emerges naturally, because the model is derived from an arbitrage trade. Along with the fair option value, a pricing model provides the option's delta, which indicates how to hedge the option's market risk exposure by taking an appropriate offsetting position in the underlying asset. Delta hedging is the mainstay of risk management for options market makers. Proper hedging requires that the pricing model be correct, of course, and also that the right volatility input be used. Even so, the delta hedging strategy that the pricing model is based upon involves continuous trading to maintain "delta neutrality" at every instant as the underlying asset's price fluctuates. In practice, this is not feasible for a diffusion process: Such a strategy theoretically entails an infinite number of transactions in the underlying over an option's life, and therefore infinite transactions costs. Moreover, financial markets do not remain open continuously, meaning that rebalancing can not always be done when it is needed. In practice, delta hedging is done approximately, with frequent periods between rebalancing trades during which the hedge will be somewhat inaccurate. The result is that a delta hedge in practice will not fully eliminate price risk.

These sources of potential model risk are easy to see, but their quantitative impact is not known. For example, while we can observe that actual stock returns are not exactly lognormal, without knowing what the true distribution is, it is difficult to judge how much inaccuracy is introduced by assuming lognormality in building a valuation model.

The object of this paper is to develop a quantitative assessment of the extent to which the different sources of model risk can be expected to affect a basic option writing strategy that might be followed by a bank or another financial institution. We conduct an empirical simulation, with and without hedging, using historical data from several important markets. Specifically, we wish to explore the following problem: If a bank or similar financial institution regularly writes standard European calls and puts and prices them using the appropriate variant of the Black-Scholes model with a volatility forecast computed optimally from historical data, what are the risk and return characteristics of the trade? More generally, what is the market and model risk exposure faced by a bank that does this transaction repeatedly over time?

The use of historical simulation to examine the performance of option trading strategies has had a long history, beginning with early papers by Galai[1977] and Merton, Scholes and Gladstein [1978, 1982]. Unlike Monte Carlo simulation in which the investigator must posit a form for the returns distribution that may be incorrect, simulation with historical data inherently employs the true distribution of returns as it would have impacted the strategy of an institutional writer of options.

In the next section, we describe the markets and the simulation strategies to be examined in detail. Section 3 discusses the various possibilities for obtaining volatility forecasts from historical data. One aspect of the research design that we emphasize throughout is that the volatility estimate should make optimal use of historical data, but only that which was available at the time the forecast would have been made. We wish to examine trading strategies as they would have been implemented. Section 4 presents the simulation results for the strategies of writing options at model prices each period and carrying the positions to expiration, either without hedging or following a delta neutral hedging policy. To see how important volatility forecast errors are to the overall result, we also compute returns that would have been obtained had the actual volatility that was realized over the option's remaining lifetime been known at each point. In Section 5 we examine the impact of variations in the strategy, including using suboptimal volatility forecasts and writing options that are either in the money or deep out of the money. Our results indicate that the various sources of model error combine to produce substantial risk exposure for the option writer, even when delta hedging of the position is actively followed. In Section 6 we consider to what extent the bank can limit the damage due to model risk by pricing options using a higher volatility than its best estimate from historical data. In Section 7 concludes.

2. Design of the Simulation

We consider a bank that is in the business of writing European calls and puts either every day, for short maturity contracts, or every month, for 2 and 5 year maturities. The options are priced at model values using standard valuation formulas of the Black-Scholes family. The required volatility input is computed from historical returns data from the market in question, using either the classical estimator for unconditional volatility with a fixed length sample of past returns, or an estimator with exponentially declining weights and all available past returns. The exact forms of the estimators are described in the next section. An important point, however, is that the volatility inputs to the models are based only upon data that would have been available at the time the calculations were needed, but they attempt to use that data optimally. In strategies involving hedging, the deltas were computed at each rebalancing point with up-to-date volatility estimates computed for those dates, as would be normal for a bank that was managing its risk properly using all of the information available to it.

In the basic simulation, we consider calls and puts with five different maturities (1 month, 3 months, 1 year, 2 years and 5 years) and two degrees of "moneyness" (at the money, with the strike set equal to the initial asset price; and out of the money, with the strike set 0.4 standard deviations away from the asset price). Basing the strike price for an out of the money option on the volatility estimate results in strike prices that are farther away from the current asset price for longer maturity contracts, but with a constant probability of ending up in the money. Under lognormality, the probability that an observation will fall more than 0.4 standard deviations below (or above) the mean is 0.34; in other words, the estimated probability that our out of the money options would finish in the money is slightly over one third. Trading volume for exchange-traded options tends to be greatest for options close to this range of strikes.

The option maturities we analyze span the range over which most options are written. For shorter term contracts of a year or less, we use daily data in the simulations. This includes computing historical volatilities from daily data, writing new contracts every day and rebalancing hedged positions daily. For 2 and 5 year maturities, we use monthly data and trading intervals.

We examine four major underlying assets from the most important asset classes upon which financial options are actively traded: a stock index (the Standard and Poor's 500), short term interest rates (3 month LIBOR), long term interest rates (the yield on 10 year U.S. Treasury bonds), and foreign currencies (the Deutschemark/dollar exchange rate). Sample periods vary, depending partly on data availability, but all begin in the early 1970's and end at the beginning of 1997. The exact data definitions and data sources are described in the Appendix.

Pricing Formulas

Although there have been numerous option models developed over the years to extend the Black-Scholes framework to new markets and to try to take account of fat tails, stochastic volatility and the model's other known shortcomings, the basic BS paradigm is still the most widely used approach for practical option valuation. Typically marketmakers will compute BS values, then add "tweaks" and subjective adjustments that they feel are appropriate for particular markets and times. We adopt the following models for our specific markets.

Equities: The original BS [1973] model for option pricing applies to a European call option on a non-dividend paying stock. The model is easily modified as shown in equation (1) to allow a dividend payout at a continuous proportional rate q during the life of the option (see Merton [1973]). We set q equal to the (realized) rate of dividend payout on the underlying index portfolio. Although traders will not know the actual future dividends at the time they must implement the model, dividend payout is quite stable and easily predicted over short intervals. However, dividend uncertainty is another source of model risk for longer term contracts.

$$C = S e^{-qT} N[d] - X e^{-rT} N[d - \sigma\sqrt{T}] \qquad (1)$$

C is the call value, S is the underlying stock index level, T is the time to expiration, N[.] denotes the cumulative normal distribution function, X is the strike level, r is riskless interest at a continuously compounded rate , σ is the estimated annualized volatility, and d is given by

$$d = \frac{\ln\frac{S}{X} + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

For put options, the continuous dividend option value is

$$P = X e^{-rT} N[-d + \sigma\sqrt{T}] - S e^{-qT} N[-d] \quad (2)$$

The call and put deltas for equity options are given by

$$\begin{aligned} Call\ delta &= e^{-qT} N[d] \\ Put\ delta &= -e^{-qT} N[-d] \end{aligned} \quad (3)$$

Interest rates: Options on interest rates require special treatment because the underlying is not a tangible asset. Although many more elaborate interest rate models have been developed over the years, for options on a single rate (unlike options on bonds, that require dealing with the whole term structure of interest rates) the variant of the BS model introduced by Black to price options on futures is widely felt to be adequate (see Black [1976]):

$$C = R e^{-rT} N[d] - X e^{-rT} N[d - \sigma\sqrt{T}] \quad (4)$$

Here, R is the underlying interest rate that the option is based on, r is the short term riskless interest rate (which is computed from the same R in the case of options on 90 day LIBOR), the other variables have the same meanings as above, and d is now given by

$$d = \frac{\ln \frac{R}{X} + \frac{\sigma^2}{2} T}{\sigma\sqrt{T}}$$

For interest rate puts,

$$P = X e^{-rT} N[-d + \sigma\sqrt{T}] - R e^{-rT} N[-d] \quad (5)$$

The call and put deltas for interest rate options are given by

$$Call\ delta = e^{-rT} N[d]$$
$$Put\ delta = -e^{-rT} N[-d] \tag{6}$$

Foreign exchange: Garman and Kohlhagen [1983] presented the variant of the BS formula for options on foreign currencies. It closely resembles equation (1), but with q set equal to the foreign riskless interest rate.

$$C = S e^{-r_f T} N[d] - X e^{-rT} N[d - \sigma\sqrt{T}] \tag{7}$$

Here, S is the level of the exchange rate in dollars per unit of foreign currency, r is the domestic riskless interest rate, r_f is the foreign riskless rate, and d is now given by

$$d = \frac{\ln\frac{S}{X} + (r - r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

For foreign exchange put options,

$$P = X e^{-rT} N[-d + \sigma\sqrt{T}] - S e^{-r_f T} N[-d] \tag{8}$$

and the call and put deltas are

$$Call\ delta = e^{-r_f T} N[d]$$
$$Put\ delta = -e^{-r_f T} N[-d] \tag{9}$$

Trading Strategy

The trading strategy we examine is to compute the model value for an option with the desired maturity and strike and to write enough contracts to produce a premium inflow of $100. This allows easy comparison of performance across contracts with different initial prices. The results of the analysis can be interpreted either as the dollar return on a $100 initial position, or as percentage returns per dollar of option premium. The proceeds are assumed to be invested, earning the current risk free interest rate in each period. For strategies that involve hedging, cash flows from all subsequent purchases and sales of the underlying asset are assumed to come out of the money market account, making the trading strategy self-financing at all times.

Risk management of an options book can range from essentially none to very sophisticated techniques that attempt to insulate the overall position against both small and large market moves, as well as changes in volatilities and other input parameters. There are three basic ways to limit overall derivatives risk. The first is simply through diversification across different markets and over time, as with stock portfolios and other risky assets. The second is through cash flow matching, such as when the bank holds offsetting long and short positions in the same option contract with different counterparties. The third is delta hedging using the valuation model, by taking positions in the underlying asset that have market exposure of the same magnitude and opposite sign as the option portfolio to be hedged.

Of these, cash flow matching offers the most precise hedge, but it is seldom possible to construct fully matched positions all of the time: If option dealers must be short options overall to satisfy the public demand to buy contracts, it is not possible for them all to be cash flow matched. This argument also applies to more complex hedges that attempt to offset the effects of convexity (gamma risk), changing volatility (vega risk), and so on.[4]

We consider the two possibilities of either not hedging at all and hoping that diversification will cause profits and losses on option writing to balance out over time, and delta hedging using the valuation model. The first simply treats an option position like any other risk asset held by the bank, while the latter trades the underlying asset as indicated by the model, in order to minimize the variability of return on each written option.

To see how much of the observed model risk is due to incorrect volatility estimates, we also examine pricing and hedging option positions using as the volatility input the actual volatility that is realized over the remaining life of the option. This presents a few difficulties, however. One of them is that when an option approaches expiration, the number of return observations remaining in its life from which to compute realized volatility obviously goes to zero. We deal with this by requiring at least 10 data points for an estimate, which are obtained by extending the period for estimation beyond option maturity when there are fewer than 10 days remaining before expiration. Another problem can occur when the strike for a deep out of the money option is set based on the volatility estimate from historical data, but this turns out to be much larger than the realized volatility over its lifetime: the strike may be set so far out of

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[4] Hedging the gammas and vegas of short option positions involves buying options. But given that there must always be a seller for every option purchased and dealers in aggregate are going to be short options, they can not all hedge these risks. Static hedging strategies will simply shift one dealer's risk to other dealers. But they will not want to increase their exposure to gamma and vega risks either, so the necessary options will be expensive to buy.

the money with the smaller volatility that the option value is extremely small. This difficulty will be discussed further in Section 5.

3. Forecasting Volatilities

One of the most important and obvious sources of error in using a model to value options is that even a correct model requires the user to input a value for the unknown volatility of the underlying from the present through expiration day. Volatility estimation error causes model risk when forecasted values are used in place of the true volatility in a pricing model. Model risk will produce mispricing of derivatives and also inaccurate hedging calculations. To give a fair analysis of model risk exposure, it is important to allow option writers to make optimal use of the information they have available in making their decisions, but not to allow them to peek into the future. This section will analyze alternative procedures for obtaining the most accurate volatility forecasts from historical data.

The most basic procedure for estimating volatility is simply to calculate the realized volatility in a sample of recent price data for the underlying and assume that the same value will apply over the future life of the derivative one is pricing. Variations on this method involve the choice of how much past data to include, periodicity (e.g., daily data versus monthly data), whether deviations are measured around the sample mean or around an imposed mean value such as zero, and whether to downweight old data.

Figlewski [1997] examined the impact on forecast accuracy of these variations, for volatility of the 3 month T-bill rate, the 20 year T-bond yield, the S&P 500 index and the Deutschemark / dollar exchange rate. A capsule summary of the findings from that study with regard to the choice of volatility estimator is as follows.

* Out of sample forecast errors of even the best method tend to be quite large.

* The forecast error for longer horizons tends to be lower than for short horizons.

* Although in practice it is common to estimate volatility from quite short historical samples, using a larger amount of past data (several times the length of the forecast horizon or more) generally gives considerably greater accuracy, except when forecasting over the very shortest horizons (e.g., less than 3 months).

* Estimating from daily data improves accuracy for short horizons (6 months or less), but for longer horizons, monthly data gives better results because it is not affected so much by transient high frequency noise in market prices.

* Since the statistical properties of the sample mean make it a very inaccurate estimate of the true mean, taking deviations around zero rather than around the sample mean

typically increases forecast accuracy.

As an example of the second point, for each month from January 1952 through December 1990 estimating volatility of the S&P 500 stock index from 5 years of past monthly data and using it to forecast volatility over the next two years gave a root mean squared error (RMSE) of 4.17% when the realized volatility averaged 14.25%; RMSE for 3 and 5 year forecasts, respectively, were 3.62% and 3.10%.

Since the reason to forecast volatility is that it is time-varying, there is a logical inconsistency in using a framework that assumes constant volatility to estimate it. Another class of volatility estimators based on past data try to model the volatility process in order to take the current state of the system into account in computing the volatility. The most common of these approaches uses the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework (see Bollerslev [1986], and Bollerslev, et al [1992]). GARCH and its variants comprise a broad set of volatility models, many of which have been developed specifically to model known features of security returns. Although GARCH has considerable success in explaining the behavior of volatility, it has several shortcomings as a forecasting tool. One is that the parameters of the GARCH model must be estimated from past data, and accurate estimation frequently requires quite a large data set. Another is that GARCH models each period's volatility as a function of the estimated volatility and the disturbance in the immediately preceding period, so the potential gains of the approach are quickly diminished when projections must be made many periods ahead.

Figlewski [1997] examined the forecasting performance of the GARCH(1,1) estimator, which is about the simplest member of the family and the one most widely used for financial data. It was found to be useful primarily for short term forecasting of stock returns volatility with daily data; for longer horizons and in different markets it did not work as well as historical volatility. And for the cases in which GARCH was the most accurate estimator, the errors were still very large. For example, RMSE in forecasting daily volatility of the S&P 500 index over a three month horizon was 5.37% relative to the average realized daily volatility of 13.29%.

The other major way to obtain a volatility estimate is directly from the market prices of traded options. The implied volatility (IV) derived from an option's market price is felt by many practitioners, and academics as well, to be the best estimate. That implied volatility is a very accurate estimate of true volatility is far from established by empirical research, however.[5] Figlewski [1997] reviews a number of empirical studies of the forecast accuracy of implied volatilities. The typical study shows that IV does contain information about future volatility, and generally more than the particular variant of historical volatility tested, but that IV is

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[5] One large study, by Canina and Figlewski [1993], found that IVs from the S&P 100 stock index options market (one of the most active in the U.S.) appeared to contain no information at all about future realized volatilities.

biased and forecast errors are substantial.

Pricing options based on IV amounts to simply pricing them the same way the market currently does. This can be useful (regardless of whether IV is an accurate forecast) when a market maker is attempting to put together a cash-flow matched position, in which a given option's contingent future cash flows are hedged by acquiring, at market prices, other options with offsetting contingencies. However, using IV as a viable forecasting tool requires a broad range of prices for traded options with different strike prices and maturities. Lacking such data, we do not examine IV here.

We therefore consider pricing and hedging options using simple volatility estimates drawn only from past returns data that would have been available at the time the forecast was made. This still leaves quite a bit of leeway regarding how much data to include in the estimate. It is reasonable to suppose that the length of the data sample that gives the most accurate forecast may be a function of the forecast horizon, so that a good one month forecast may be calculated from a few months of recent returns data, while the best five year forecast will require looking much further back in time. Figlewski [1997] examines a forecast procedure that at each date computes the unconditional volatility (i.e., as if it were a fixed constant) but optimizes over the amount of past data to include, so as to achieve the minimum RMSE out of sample predictions. In the spirit of ARCH-family nomenclature, this technique is dubbed, somewhat tongue-in-cheek, Optimized Unconditional Conditional Heteroskedasticity (OUCH).

The nature of the estimation error in predicting volatility from our historical data samples with an OUCH model is illustrated in Tables 1A and 1B. We examine daily and monthly data on 3 month U.S. dollar LIBOR, the 10 year T-bond yield, the S&P 500 stock index and the Deutschemark / dollar exchange rate. For each market, several different forecasting horizons are examined, and for each, we try three historical sample lengths and also a forecast using exponentially declining weights.

To explain the details of the procedure, let us focus on the single example of forecasting the volatility of 3 month LIBOR over a 24 month horizon using 24 months of historical data, shown in Table 1B. Our data sample begins in January 1971. Since we need up to 5 years of past data to compute some of the estimates and we want to be able to compare the performance of all of the methods over the same historical time period, the first forecast date is January 1976. In this case, continuously compounded changes for the observed interest rates from the last 24 months are computed by taking the first differences of their logarithms. Rather than calculating the standard deviation around the sample mean, which is generally a very inaccurate estimate of the true mean, we impose zero as the mean change in rates. The variance is then just the average of the squared log changes. This is annualized by multiplying the monthly variance by 12; and taking the square root gives the volatility. This estimate is then used as the volatility forecast going forward from January 1976. The historical volatility over the past 24 months just calculated becomes the first forecast for both the 24 month horizon and the 60 month horizon.

Next we compute the realized volatility over the 24 month period from January 1976 to January 1978, also constraining the mean to zero. The difference between the forecast and the realized volatility becomes the forecast error for January 1976. The forecast date is then advanced one month and the procedure is repeated. This continues until December 1991, the last date for which our data sample allows calculation of the realized volatility over a 60 month forecast horizon. The series of volatility forecast errors are then squared and averaged, and the square root is taken to produce the root mean squared forecast error of 13.3%. This is the measure of forecast accuracy for the strategy of using 24 months of past data to predict future LIBOR monthly volatility over a 24 month horizon.

We note that the forecast errors will not be serially independent, since the volatilities calculated for consecutive dates are computed from almost exactly the same data points. Lack of independence will not bias the estimated RMSEs, although it would cause inconsistency in an estimate of the standard error of the RMSE. We think of this procedure as a way to assess the impact of estimation error for a financial institution that writes options every period (i.e., daily or monthly), basing pricing and hedging strategy on volatility values estimated from past data in a standard way. Lack of independence will produce serial correlation in the pricing errors, which will show up in the form of runs of losing and winning months. To give an idea of how this impacts performance, in later tables we report both the worst single month and the worst full year for the option writing strategy.

The same procedure is used to fill in RMSE values for all combinations of market, historical sample and forecast horizon. The sample period labeled "All available" uses all past data back to the beginning of the sample as of each date. Thus, unlike the other methods, the amount of historical data in this volatility estimate grows over time, from 5 years on the first date to 20 years or more on the last. The table shows that this is an effective strategy in three of the four markets we consider.

The "All available" estimate uses every past observation weighted equally. However, it is generally felt that recent observations are more meaningful than ones from the distant past. A relatively easy way to take account of this is to weight each data point in inverse proportion to its age. The exponentially declining weight forecast is computed as in equation (10), where $0 < w \leq 1$ is the weighting factor and R_t is the log price relative from date t.

$$\sigma_{forecast\ at\ T} = \left(\frac{\sum_{t=1}^{t=T} w^{T-t} R_t^2}{\sum_{t=1}^{t=T} w^{T-t}} \right)^{1/2} \qquad (10)$$

In the limit, equation (10) includes an infinite number of past observations, but with those from the distant past having infinitesimally small weights. Thus, depending on the value of w, the weighted forecast may be largely determined by data from the recent past. One way to measure the effect of this weighting scheme is by the mean lag, which is the weighted average date for the sample points in the estimate. The mean lag can be computed simply as

$$Mean\ \ lag \ = \ \frac{1}{1 \ - \ w} \tag{11}$$

For example, the average w value for forecasting LIBOR daily volatility over 1 month was computed to be w = 0.973. From equation (11) this w corresponds to a mean lag of 37 days. Note that the mean lag figures shown in Table 1B are in months.

In the results shown here, the weights were computed for each date by analyzing only historical data that would have been available at the time, with the optimality criterion being out of sample RMSE. For each date, this required dividing the available historical data into an estimation sample and a forecast sample. The procedure we have adopted is somewhat arbitrary, but represents one plausible way for such a computation to be done.

As an example, consider the 24 month forecast horizon using monthly observations. To allow multiple past forecast horizons, on date T we compute realized volatility over 12 overlapping 24-month periods, {T-25 to T-1, T-26 to T-2,..., T-36 to T-12}. Given a trial value for w, for each of those periods we compute a volatility forecast using equation (10) with all available data prior to the beginning of that period. This produces 12 (unfortunately overlapping) forecasts and forecast errors from which the RMSE is calculated. We then search over values for w to obtain the value that would have minimized RMSE. That is the w used on date T to forecast volatility over T+1 to T+24. We then advance a month and repeat the process to obtain a weighted volatility forecast for the period T+1 to T+25 based on a reestimated w value, and so on. In each case, we impose the constraint w ≤ 1.0.

The difficulty arises at the beginning of the sample. To the extent possible, we would like to use an estimation sample of at least five years and a forecast sample allowing multiple periods of length equal to the forecast horizon we are trying to optimize for, so that a meaningful RMSE can be computed. That is not feasible in every case. To give a larger historical sample for the first forecast dates, we set w = 1.0 for the first year with the daily data and for the 24-month horizon, and for the first 4 years with the 60 month horizon. This still can leave only 3 years of sample data to compute the volatility for the first forecasts that go into the RMSE we are testing to find the optimal w. This problem only affects the beginning of the sample, but it is inherent in the use of an exponentially declining weight forecast when the weight must be obtained by examining past data.

Table 1A presents forecast results for daily volatility over horizons of 1, 3, and 12 months using daily historical data. Table 1B does the same thing for monthly observations to forecast over 2 and 5 year horizons. In the daily table, for convenience in estimation a "month" is defined to be 21 trading days in all cases. Notice that the samples span slightly different periods. In particular, it is only necessary to hold out 12 months of daily data at the end of the sample for post-sample forecasting, rather than 5 years as in the monthly table.

In each column, the shaded figure indicates the minimum RMSE historical sample size (or in some cases, the exponentially weighted forecast). In the next sections, we will assume the volatility estimate to be input to the model on each date was computed using this method. This is the one place where we have not strictly adhered to the requirement that all decisions be based only on information that would have been available at the time: the analysis shown in these tables indicating which horizon was best uses data from the entire sample. Thus, early in the sample, without knowledge of subsequent returns behavior, an actual bank following the procedure we have outlined might not have used the minimum RMSE historical sample length, and would have made less accurate forecasts than what we are assuming here.

One striking result is how large the forecasting errors are. For example, on average the forecast of 3-month LIBOR volatility over the next 24 months based on the last 24 months had an RMSE of 13.3 percent. This is very large relative to a mean realized volatility of 25.4 percent. Roughly speaking, this means that about a third of the time, the predicted volatility would be more than 50% above or below the true value. LIBOR is actually the worst case, but forecast errors are substantial for all of these markets.

The results shown in these tables are consistent with those reported in Figlewski [1997] for different sample periods and different markets. For monthly data, it is generally the case that the best estimates come from using the largest possible historical sample, but for daily observations and shorter forecasting horizons, performance is better if the historical sample is several times as long as the horizon, but not too long. Surprisingly, forecasting accurately over longer horizons seems to be easier than over shorter ones.

One interesting feature of these results is that (annualized) volatility for daily data appears to be substantially lower than for monthly data. This may be due partly to the somewhat different sample periods. It also may be a result of short term positive serial correlation in the daily data series. If price changes are not independent over time, estimated volatility will be affected. Positive autocorrelation, which occurs when observed prices adjust to new information with a lag over short intervals, reduces estimated volatility. This problem largely disappears with longer differencing intervals, which is the reason to use monthly rather than daily data for longer horizon forecasting.

4. The Risk and Returns to Option Writing Using Forecasted Volatilities

In Tables 2A - 2D, we examine the impact of volatility forecasting errors on a bank or financial institution that writes options each period (either every month or every day). In each case, enough options are sold to produce $100 of premium income, so the performance figures may be interpreted as percentages of the initial option price. Results are presented for at the money and slightly out of the money calls and puts for the five daily and monthly forecast horizons examined in Tables 1A and 1B. As described above, the out of the money options are assumed to have strike prices that are 0.4 standard deviations away from the current asset price level (in terms of the estimated volatility). The top portion of each table gives the results for writing options of the specified type (e.g., at the money calls) each day, using a volatility forecast computed from daily historical data, and the bottom portion does the same for the longer horizons and monthly volatilities.

Tables 2A-2D look at the strategy of selling the options at their model values, investing the proceeds at the risk free interest rate, and simply holding the short position until maturity without hedging. The left side of each table uses forecasts for the given horizons from the lowest RMSE methods in Tables 1A and 1B. The results therefore reflect model error due both to inaccurate volatility inputs and to errors in the models (such as a failure to fit tail probabilities accurately). The right side of each table shows the results if the same options are priced using the realized volatility over the option's life as the input to the model. This removes the volatility estimation error, leaving only the effect of inaccuracy in the valuation model itself, along with random errors due to the finite sample.

For each market and horizon, the first two columns give the mean return and standard deviation of the strategy. These are reported on a "per trade" basis, i.e., not annualized. For example, writing $100 worth of 1 month S&P 500 Stock Index calls lost on average $ -21.52 over the month, with a standard deviation of $158.57 across months.

Since there is no hedging, the theoretical mean return to option writing should be a function of the expected value of the change in the underlying asset. For options on the S&P stock index, this should be positive and well above the risk free interest rate for calls, and negative for puts. Over the long run, stocks have averaged returns between 8 and 9 percent above Treasury bills, and as leveraged instruments, options should have larger risk premia than the underlying stock index. Thus, a call writing strategy for the S&P 500 index should lose money on average, as it does here, while writing puts should be profitable, because they have negative betas. For the other three markets, there is no presumption that the expected change will be either positive or negative, so our prior expectation is that both call and put writing with reinvestment of the proceeds at the riskless rate should break even on average.

One thing that is clear in these results is that without hedging, standard deviations are very large, and it does not make much difference whether the volatility is known or just forecasted. Indeed, since the strategy simply amounts to taking a bet that the underlying will not move too far in the wrong direction over the option's lifetime, the results are dominated by what these markets actually did during this period of approximately 20 years. Although there were obviously both up and down movements in each market, the overall trend for stocks was strongly upward. It was downward for short term interest rates like LIBOR, and fairly steady to slightly down for 10 year Treasury yields and for the Deutshemark (DM) exchange rate.

Thus, the at the money stock index calls that were written had a strong likelihood of ending up in the money which, without hedging, produced large losses of 20% to 50% on average for the shorter term contracts and more than 170% of the premium received for the longest term calls (all of which finished in the money). Knowledge of the true volatility that would be realized over the option's life did not improve profitability. Indeed, mean returns for index calls were a little worse and the standard deviations were about the same. For other markets, the "write and hold" strategy would have produced losses on average in some cases and profits in others, but risk exposure as measured by the standard deviation was quite large in every case, generally well over 100% of the initial premiums. Out of the money options, not surprisingly, showed substantially greater standard deviations in Tables 2B and 2D.

Another dimension of risk exposure is the single worst loss registered by a given strategy. The column labeled "Worst Case" gives those figures and shows that writing options unhedged can produce extremely serious losses. In most cases, the worst single trade for a given market lost more than 10 times the amount of premium income received. The most disastrous outcome of all was from writing out of the money puts on the DM, which produced a loss nearly 300 times larger than the initial premium received for 1 year options written on 6/28/77. Unlike the mean and standard deviation figures, in many cases the worst outcomes were less bad when the true volatility was known than when the historical estimate was used.

A problem alluded to above is that volatility forecasts estimated from long historical data samples do not change rapidly from day to day or from month to month. This means that there may be strings of losing trades, in which the options written over a number of periods all go bad in sequence. The final columns in each row show the worst calendar year and the mean returns for options written in that year. For example, 5 year at the money S&P call options written every month during calendar 1982 lost on average 568% of the initial premium. In other words, for the $1200 of premium received from writing calls each month in 1982, without hedging the writer would have lost a total of 12 x 568.68 = $6825.84 in the year they matured.

These results show clearly that the strategy of writing and holding option positions without hedging entails very large risk exposure, even when pursued consistently over a long period. Diversification of the outcomes over time is not sufficient to control risk to reasonable levels.

Tables 3A - 3D give the results when the written options positions are delta hedged over their full lifetimes. For options written on the S&P 500 or on the DM, we assumed hedging was done using the underlying cash instrument, either the S&P stock portfolio or the appropriate number of Deutschemarks. The hedge ratio adopted was the model delta from equation (3) or

(9), and the quantity traded was set to produce “dollar equivalence” in the hedge.[6]

For the two interest rate contracts, a different hedging approach was required. For instance, it is not feasible to hedge changes in future LIBOR (i.e., the level of LIBOR on the date the option’s payoff will be determined) by trading actual securities. Instead, we assumed the interest rate options would be hedged using futures contracts: 90 day Eurodollar futures traded on the Chicago Mercantile Exchange, and 20 year Treasury bond futures traded at the Chicago Board of Trade. The T-bond contract was first introduced in 1977 and the Eurodollar contract did not begin trading until 1982, so the sample had to be trimmed at the beginning.

Hedging long term option contracts with futures presents several important questions of hedging tactics. Ideally, the futures expiration should be set equal to the maturity date of the option, but this was not feasible for longer term options, because futures did not always exist with long enough maturities, and when they did exist, they were often highly illiquid. Also, in the case of the T-bond contract, any open long position (such as would be taken to hedge written put options on the Treasury yield) that was carried into the contract’s expiration month could potentially result in undesired delivery of the underlying bonds. We therefore adopted the following procedures.

Options on the Treasury bond yield were hedged in all cases with the nearest to expiration future (which has the greatest liquidity). These positions were assumed to be closed out and rolled over into the next expiring contract on the last trading day prior to the nearby contract’s expiration month. Over time, the Eurodollar futures contract has developed liquidity at more distant maturities and trading has extended to quite long term contracts. This was not true at first, however. We have therefore assumed that an option with less than 1 year to expiration was hedged with the nearest future that matured after its expiration date, but longer term options were always hedged with the 1 year maturity future. This can translate to a large maturity mismatch for 2 and 5 year options, and may partly account for the poor hedging performance seen here.

Comparing Tables 3 with Tables 2, it is clear that delta hedging greatly reduces risk exposure for these written options. For example, the standard deviation of the return on 3-month at the money S&P 500 calls drops from 159.51 to 24.74. In virtually every case, hedging reduces the standard deviation, the size of the mean, and the magnitude of the worst cases. Nevertheless, the standard deviations of the returns remain sizable, and quite large--over 100%--in some cases. The worst losses still were many times the initial premium received, particularly for the out of the money contracts.

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[6] “Dollar equivalence” means that the amount of the hedge instrument held is set so that when the price of the underlying asset changes, it (theoretically) produces an offsetting cash flow exactly equal in dollar magnitude and opposite in sign to the change in the value of the item being hedged.

The interest rate options hedged with futures contracts retained more of their variability than did the S&P and DM options. Several factors might be contributing to this. First, there is the basis risk that affects futures prices. Futures prices are tied to the price behavior of their underlying assets, but the relationship is flexible, so that mispricing can arise and persist for some time. The fact that futures may be mispriced at the time a hedge must be rebalanced adds variability to the overall return. Such basis risk is likely to be greater when there is a mismatch between the item being hedged and the future. That happens here because of the need to use futures with different maturities from the options they are hedging, and also, in the case of Treasury bonds, because the future is based on a 20 year bond (or, more precisely, on the cheapest to deliver bond, that might have up to 30 years to maturity) while it is the 10 year rate that determines the option payoff. Thus, changes in the slope of the yield curve will produce basis risk in the hedged position. Finally, there is a large amount of empirical evidence that interest rates follow more complex processes than the standard diffusions that the Black model allows for, suggesting that model inadequacy may well be greater for the interest rate options.

Tables 3A -3D show how much estimation error on the volatility input affects the variability of hedge returns. For both S&P and DM options, knowing the volatility would have reduced the standard deviation substantially. This did not necessarily hold true for the interest rate contracts. While hedging with futures did reduce position risk for LIBOR and T-bond yield options relative to not hedging at all, the improvement was generally rather limited.

The results illustrate that writing options and delta hedging the positions using the most common valuation models involves exposure to considerable risk, much of it “model” risk. Hedging is much better than not hedging, but a great deal of risk still remains. Note also that we have made no attempt to take into account any transactions costs of running a hedge strategy over long periods. In practice, there would be brokerage fees and market impact costs (e.g., the bid-ask spread) that would increase with the amount of rebalancing needed to maintain delta neutrality. On the other hand, we have also not included any “markup” on the written option contracts. A bank writing options to satisfy its customers should not price them exactly at their model values, since in theory they should only break even at those prices. It would be normal to raise the price above the model values, which would cover many of the mean losses in these tables. We will look at that strategy in more detail in Section 6.

5. Variations on the Theme

The previous section looked at writing at the money and slightly out of the money options, which we expect to be the ones with the greatest activity, and assumed that the volatility input to the model was the historical estimate with the best forecasting power. We will now briefly explore some “variations on the theme:” writing deep out of the money or in the money contracts, and using suboptimal volatilities.

We have defined an "out of the money" contract to be one with a strike price 0.4 standard deviations above the current asset price for a call (or below, for a put). For a lognormal distribution without drift, there is about 0.34 probability that such an option will finish in the money. Continuing this idea, we now define "in the money" and "deep out of the money" options as being 0.4 standard deviations in the money, or 1.0 standard deviations out of the money, respectively. The equivalent probabilities of finishing in the money are about 66% and 16%, respectively.

Volatility forecasting for option traders is widely thought to be an art as much as a science. There is a common feeling that since volatility changes over time, it is better to eliminate old data from the calculation. Volatility estimates provided by online derivatives services are typically computed from 3 or 6 months of past daily returns, and often from as little as one month. Using exponentially declining weighted forecasts also has the effect of placing most of the weight on the recent past. For example, the daily volatilities presented by J.P. Morgan's RiskMetrics system use data from the last 25 trading days, with an exponential decay factor of w = 0.94. From equation (11), this means the mean lag in the estimator is only about 17 days, and it contains no information more than 25 days old.[7]

The results shown in Tables 1A and 1B, however, suggest that using such a short historical sample leads to larger RMSE forecast errors than would be achieved with more past data. For the one-month horizon, the most accurate volatilities come from at least several months of data, while for the longer horizons examined in Table 1B, the most accurate volatilities used all available historical volatilities in three markets out of four, and the fourth still gets the most accurate estimates using 60 past months. This suggests that many option writers may be pricing and hedging their positions using volatilities calculated from less than the optimal amount of historical data.

Results from classical statistics show that when the underlying asset follows a diffusion process, much greater accuracy in estimating volatility can be obtained by using high frequency data. This suggests that if both daily and monthly data are available, it is much better to use daily data. On the other hand, because the data are prices recorded in financial markets, they may actually deviate quite far from true diffusions at very short intervals. In markets with a bid-ask spread, transaction prices tend to show high negative autocorrelation due to "bid-ask bounce." Stock indexes observed daily may show spurious positive serial correlation because of infrequent trading of less liquid stocks. And in still other cases, like LIBOR, changes at the shortest intervals are not diffusions at all; the series either remains constant or jumps instantaneously by a relatively large discrete amount (e.g., 1/16th or 1/8th of a percent).

These observations suggest that unless the forecast horizon is very short, such that the essentially spurious price effects due to trading noise may nevertheless be relevant

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[7] See "RiskMetrics" [1995].

determinants of option payoffs, a volatility estimator based on daily data may actually give less accurate values than one with a longer observation interval. In other words, if one is writing 2 year contracts, it may well be better to estimate volatility from monthly data than from daily data over the same historical period. We do not provide evidence of this principle here, but we have incorporated it into the analysis by using monthly data for the longer horizon contracts and daily data for the shorter ones. Figlewski [1997] finds that in using five years of historical data to forecast volatility over 24 months, daily observations increase RMSE by about 5 to 10 percent above that achieved with monthly data.

Tables 4A and 4B present results on these issues, using three alternative volatility estimators to implement a strategy of writing and delta hedging calls and puts with four different strikes and two maturities, for the markets we have been examining. The mean and standard deviation of the per trade return are shown, along with the average return per trade in the single worst year for the strategy. For each market and maturity, the first line of results corresponds to using the realized volatility over the option's remaining lifetime at all times, thus eliminating model risk due to the volatility forecast. The next line shows the results for the best volatility predictor for that horizon, based on out of sample performance (i.e., the shaded cells from Tables 1A and 1B). For exponentially weighted forecasts, the mean lag in days corresponding to the average decay factor for that market is given in parentheses.

The third line presents results based on suboptimal volatility forecasts: 5 years of daily data for 1 month options, and 3 months of daily data for 2 year contracts. Thus for the short term options, we use a historical sample that is substantially longer than indicated, while for the longer term contracts, we use "too little" data, in the form of frequent observations drawn from a very short sample period.

These tables allow easy comparison across several important dimensions. Comparing calls versus puts, we see that while results are obviously different, there is no clear distinction in the character of the outcomes. Whether call writing or put writing was more profitable, or riskier, varied from case to case. It is worth noting in passing, that with delta hedging, the profitability of the trade can be expected to depend on the behavior of volatility as much or more than on market direction. If volatility has been underestimated, hedged writing of both calls and puts should lose money, as illustrated by the case of trading 2 year S&P 500 options using volatilities computed from the last three months of daily data.

Comparing across strikes for a given market shows that even hedged writing of out of the money options, especially those that are deep out of the money, entails considerable risk. Standard deviations for these contracts are nearly all over 50% of the initial premium, and much greater in some cases. Some of the truly huge numbers, like those for DM puts, are somewhat artificial, however. The strike levels for these options are set in terms of standard deviations based on the "best" volatility forecasts. In cases where the "bad" estimator predicts much lower volatility, these options appear to be very far out of the money. Their model values are then extremely low, meaning that very large numbers must be written to produce

$100 of premium income, and losses are enormous if even a few cases end up very far in the money.[8]

Model risk due to inaccurate volatility estimates is a significant contributor to overall risk, as evidenced by the differences between the first and second lines. For the interest rate contracts, even knowing the true volatility that would occur did not hold the risk down--standard deviation exceeded 100% in the majority of cases. Risk was relatively more manageable for the equity and currency options. Comparing the results for the optimal and the suboptimal volatility predictors, one sees that for the most part, using too much past data for the 1 month options did not hurt performance that much, but using too little data for the 2 year contracts was a greater problem.

Finally, it is interesting to note that the per trade returns did not seem to be much riskier for the long term contracts than for the shorter ones. This is odd, but it is consistent with the equally counterintuitive forecasting results shown above and in greater detail in Figlewski [1997], that volatility forecasts are more accurate for longer horizons than for shorter ones.

6. Writing Options with a Volatility Markup

The results so far indicate that a regular option writing program would entail a large amount of risk even with delta hedging of the market exposure. Moreover, the pricing models are based on the principle that a perfectly hedged position involving an option and its underlying asset should earn the riskless interest rate. But, of course, no financial institution would enter into this business if it was expected to earn only the T-bill rate. The normal procedure when writing an over the counter option contract is to obtain the best available volatility forecast, then increase it by some suitable amount in pricing the option. The higher volatility produces a higher option price, which is expected to compensate the writer for bearing the various kinds of risks and to provide an expected profit above the riskless rate.

Tables 5A and 5B examine the volatility markup strategy in our markets. The tables show results for two maturities, daily writing of 3 month options and monthly writing of 2 year contracts. The premiums are computed by increasing the volatility input from the optimal historical model by 10%, 25%, or 50% (i.e., from σ to 1.1 σ, 1.25 σ, or 1.5 σ). Hedging, however, is always done using the unadjusted forecasts. We report the mean and standard deviation of the returns, the percentage of positions that lose money, and the average size of the loss for a losing position.

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[8] It is worth noting that this type of problem is unlikely to be significant in practice, because any bank actively writing options will be aware of how their competitors are pricing similar contracts. They would notice if their prices were well below the market because of an unusual volatility estimate.

Boosting the volatility clearly helps to increase the mean returns to option writing. Standard deviations are only slightly affected, but the fraction of trades that lose money is decreased rather sharply in most cases. For example, writing 3 month at the money S&P 500 index calls lost money 46.8% of the time when options were priced at their estimated Black-Scholes values, but increasing the volatility input by one quarter lowered the loss percentage to 16.0. Interestingly, while each formerly losing trade should lose less after the volatility boost, in many cases the average loss became worse. While only a much smaller fraction of the trades ended up losing, the losses that remained were big ones.

Thus, boosting the volatility input turns a very risky trading strategy that exposes the writer to very large potential losses into one that may allow quite substantial profits on average and relatively few losses. However, the losing trades still entail large losses. For example, for deep out of the money S&P 500 calls at the 2 year horizon, boosting volatility by 25% cut the fraction of losing trades from 37.3% to 15.4%, but those that did lose money lost on average almost 140% of the initial premium. This pattern is perhaps even more striking for writing put options. Table 5B shows that put writing was profitable in most cases, and raising the volatility input produced handsome profits. However, the average losses for those positions that did lose money were quite severe.

7. Conclusion

We have examined a basic trading strategy which we believe fairly closely reflects institutional practice at many banks and other financial institutions that are engaged in the business of writing derivatives contracts. Obviously, the most sophisticated derivatives trading firms can be expected to employ more finely developed models and risk management tools than what has been considered here. It is not clear, however, that any large improvement in volatility forecasting is possible beyond the "OUCH" technique we analyzed, as the evidence from the literature discussed in Figlewski [1997] seems to show.

The results we have obtained indicate that model risk in trading and hedging derivatives is quite large. Of the three risk management strategies discussed at the outset, our analysis makes clear that simply writing and holding options as if they were ordinary risk assets, entails a very large risk exposure, with standard deviations of per trade returns well over 100% of premium income in many cases. Cash flow matching, if it can be done precisely, is clearly a sound risk management strategy, since it removes the impact of model risk entirely, and market risk, as well. The entire risk in a cash flow matched position is essentially credit risk: the risk that one of the counterparties in the matched position will default on a losing trade and leave the hedger exposed. Cash flow matching, however, is not a strategy that can be followed generally, as we have argued, because the nature of the derivatives business is that the public wants to be long options, so that the dealer community must be short on balance. Dealers as a group must hold exposed, unmatched, option positions.

This leaves delta hedging based on valuation models implemented with forecasted volatilities as the only viable trading and risk management strategy for most options writers. Our results show that this can be expected to involve considerable risk exposure. Two important components of an overall risk management strategy for a derivatives book, therefore, should be use of the best pricing models and volatility estimators, and also diversification of (delta hedged) risk exposures across a variety of derivatives markets and instruments, with the hope that a "worst year" for one asset class may be mitigated by good years for others. A third component is simply to charge higher prices than the model indicates for the options that are written. Our results suggest that increasing the volatility input by a quarter to a half would substantially increase mean returns and reduce the fraction of losing trades. However, the worst losses to the strategy would remain very large--there would just be fewer of them.

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Appendix

Data Sources

This Appendix describes the sources and handling of the data series used in the study.

Standard and Poors Stock Index: The data for 1/4/71 - 12/31/96 come from CRSP. The daily dividend payout on the index portfolio is constructed as the difference between the dividend inclusive return series and the price change series. Monthly data are constructed from the daily series.

90-day US Dollar LIBOR: For the period 1/4/71 - 1/1/75 the data are from the Harris Bank, and are weekly. From 1/2/75 - 12/31/96 the data are daily, from Datastream (series name: EURO-CURRENCY (LDN) US$ 3 MONTHS - MIDDLE RATE ECUSD3M).

10 Year Treasury Yield: The source is the Constant Maturity 10 Year Yield series from the Federal Reserve H.15 report. Data come directly from the Federal Reserve for 1/4/71 - 2/2/96. The H.15 data were obtained from Bloomberg for 2/3/96 - 12/31/96.

Deutschemark / Dollar Exchange Rate: Data for 1/4/71 - 12/1/95 are from the Federal Reserve H.10 report. From 12/2/95 - 12/31/96 the source is Datastream (series name: DMARKE$).

Futures Prices: Daily closing prices for the 90 Day Eurodollar futures contract at the Chicago Mercantile Exchange and the 20 year U.S. Treasury Bond futures contract at the Chicago Board of Trade were obtained from the Futures Industry Association.

U.S. Riskless Interest Rate: The 90-day Eurodollar interest rate was used, after conversion to the equivalent continuously compounded rate.

German Riskless Interest Rate: The EuroDeutschemark 3 month rate was used, after conversion to the equivalent continuously compounded rate. For 1/4/71 - 12/31/80 the source was the Harris Bank. From 1/1/81 - 12/31/96 the data are from Datastream (series name: EURO-CURRENCY (LDN) D-MARK 3 MONTHS - MIDDLE RATE ECWGM3M).

Table 1A

Forecast Accuracy of Volatilities Estimated from Historical Data

Daily Data

The table shows the root mean squared forecast error for annualized volatility calculated from daily data around a mean of zero, for different forecast horizons and historical sample lengths. Also reported is the performance of exponential weighting across all past data, in which the optimal weight on each date is the one that would have minimized the forecast error for that forecast horizon over the historical sample available at the time. One "month" is defined as 21 trading days. Shading indicates the minimum RMSE estimation method.

S&P 500 Stock Index

December 30, 1975 - January 3, 1996

Months in Sample	Forecast Horizon (Months) 1	3	12
3	7.2%	7.2%	7.1%
12	7.5%	7.1%	6.8%
60	7.8%	7.2%	6.5%
Exponential wgts	7.0%	6.7%	6.5%
Average weight	0.979	0.986	0.990
Mean lag (days)	47	73	103
Average Realized	12.9%	13.2%	13.7%

US $ London Inter-Bank 3 Month Rate

November 1, 1979 - January 12, 1996

Months in Sample	Forecast Horizon (Months) 1	3	12
3	10.2%	9.3%	8.4%
12	10.2%	8.8%	8.1%
60	13.0%	11.5%	10.3%
Exponential wgts	9.8%	8.7%	8.0%
Average weight	0.973	0.984	0.987
Mean lag (days)	37	64	80
Average Realized	20.8%	21.5%	21.5%

10 Year Treasury Bond Yield

January 26, 1976 - January 5, 1996

Months in Sample	Forecast Horizon (Months) 1	3	12
3	4.6%	4.4%	4.4%
12	4.8%	4.2%	4.0%
60	5.7%	5.1%	4.6%
Exponential wgts	4.7%	4.6%	4.7%
Average weight	0.976	0.986	0.992
Mean lag (days)	41	70	132
Average Realized	12.7%	13.0%	13.5%

Deutschemark Exchange Rate

January 22, 1976 - January 10, 1996

Months in Sample	Forecast Horizon (Months) 1	3	12
3	3.9%	3.6%	3.3%
12	3.8%	3.2%	2.8%
60	4.0%	3.3%	2.5%
Exponential wgts	4.0%	3.3%	2.8%
Average weight	0.981	0.994	0.995
Mean lag (days)	52	160	203
Average Realized	10.0%	10.3%	10.5%

Table 1B

Forecast Accuracy of Volatilities Estimated from Historical Data
Monthly Data

The table shows the root mean squared forecast error for annualized volatility calculated from monthly data around a mean of zero, for different forecast horizons and historical sample lengths. Also reported is the performance of exponential weighting across all past data, in which the optimal weight on each date is the one that would have minimized the forecast error for that forecast horizon over the historical sample available at the time. Shading indicates the minimum RMSE estimation method.

S&P 500 Stock Index
Jan 1976 - Dec 1991

Months in Sample	Forecast Horizon 24	Forecast Horizon 60
24	5.9%	5.4%
60	4.9%	4.5%
All available	4.0%	3.2%
Exponential wgts	4.2%	4.1%
Average weight	0.976	0.987
Mean lag (months)	42	79
Average Realized	15.4%	15.2%

US $ London Inter-Bank 3 Month Rate
Jan 1976 - Dec 1991

Months in Sample	Forecast Horizon 24	Forecast Horizon 60
24	13.3%	11.9%
60	12.6%	10.5%
All available	15.2%	13.5%
Exponential wgts	13.7%	12.8%
Average weight	0.898	0.942
Mean lag (months)	10	17
Average Realized	25.4%	25.3%

10 Year Treasury Bond Yield
Jan 1976 - Dec 1991

Months in Sample	Forecast Horizon 24	Forecast Horizon 60
24	5.4%	5.0%
60	4.7%	4.2%
All available	4.4%	3.4%
Exponential wgts	5.3%	3.7%
Average weight	0.971	0.991
Mean lag (months)	35	107
Average Realized	15.2%	15.7%

Deutschemark Exchange Rate
Jan 1976 - Dec 1991

Months in Sample	Forecast Horizon 24	Forecast Horizon 60
24	3.5%	2.8%
60	2.5%	1.3%
All available	2.4%	1.4%
Exponential wgts	2.7%	1.5%
Average weight	0.977	0.987
Mean lag (months)	44	77
Average Realized	12.3%	12.3%

Table 2A
Return and Risk in Writing Options with Estimated Volatilities: At the Money Calls

The table reports the performance of a strategy of writing unhedged at the money call options each period for several maturities. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500 Stock Index	1 month	-21.52	158.57	61%	8/12/82	-1035.20	1975	-525.52	-29.17	157.54	61%	1/15/91	-627.15	1975	-528.51
	3 months	-33.57	159.51	68%	8/11/82	-1077.77	1975	-334.20	-38.47	155.55	68%	8/11/82	-606.38	1975	-387.99
	1 year	-56.22	164.20	76%	7/28/82	-688.87	1982	-344.38	-78.25	173.38	76%	12/8/94	-706.15	1995	-340.70
3-month US$ LIBOR	1 month	2.97	163.37	43%	2/1/94	-1140.84	1994	-184.31	3.33	144.69	43%	4/8/94	-683.59	1994	-165.08
	3 months	5.12	164.57	43%	2/14/94	-837.47	1994	-307.20	-6.42	174.47	43%	7/21/87	-780.49	1994	-308.59
	1 year	-20.80	247.74	38%	12/17/93	-1268.64	1993	-456.56	-42.47	288.61	36%	1/18/94	-1310.51	1993	-479.98
Ten-Year Treasury Yield	1 month	-30.89	215.31	50%	9/21/79	-2189.14	1977	-218.78	-22.94	167.62	50%	9/21/79	-635.42	1977	-212.98
	3 months	-40.16	221.98	53%	8/7/79	-1473.74	1979	-386.28	-35.09	188.74	53%	2/29/84	-735.01	1977	-391.65
	1 year	-70.44	274.19	50%	2/20/79	-1860.33	1979	-676.18	-47.16	214.96	50%	2/20/79	-858.14	1977	-588.72
Deutschemark Exchange Rate	1 month	5.36	155.32	47%	2/26/91	-1132.42	1991	-97.77	1.96	146.66	47%	2/26/91	-759.29	1983	-67.04
	3 months	6.91	162.82	45%	2/8/91	-1184.26	1991	-168.60	6.56	147.97	45%	1/29/91	-695.17	1984	-100.24
	1 year	15.77	146.65	46%	8/21/92	-704.32	1980	-327.21	20.35	131.42	46%	8/24/92	-555.63	1980	-254.26
Monthly Observations															
S&P 500 Stock Index	2 years	-59.96	142.93	92%	Sep-85	-602.51	1985	-369.70	-75.49	148.31	91%	Sep-85	-528.10	1984	-320.26
	5 years	-172.46	182.41	100%	Jul-82	-883.74	1982	-568.82	-174.06	182.77	100%	Jul-82	-874.96	1982	-562.95
3-mo. US$ LIBOR	2 years	-16.14	250.64	36%	Feb-93	-836.53	1993	-749.55	-24.97	261.14	35%	Feb-93	-872.04	1993	-784.91
	5 years	144.01	9.74	5%	May-85	108.51	1990	129.04	143.24	12.53	5%	May-85	78.54	1990	129.04
Ten-Year Treasury Yield	2 years	-60.28	307.71	43%	Sep-79	-1292.01	1979	-862.62	-13.62	204.94	43%	Feb-78	-668.89	1977	-562.29
	5 years	37.54	250.87	29%	Aug-77	-1096.42	1977	-668.10	80.17	155.03	29%	Aug-77	-716.22	1977	-393.82
Deutschemark Exchange Rate	2 years	37.05	121.94	43%	Dec-91	-401.84	1980	-221.30	38.79	119.19	43%	Dec-91	-381.12	1980	-214.64
	5 years	88.49	135.57	27%	Feb-80	-426.01	1980	-213.79	89.96	133.10	27%	Feb-80	-405.27	1980	-210.84

Table 2B

Return and Risk in Writing Options with Estimated Volatilities: Out of the Money Calls

The table reports the performance of a strategy of writing unhedged out of the money call options each period for several maturities. The strike price is always set 0.4 standard deviations above the current spot. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500 Stock Index	1 month	-24.22	218.35	44%	8/12/82	-1661.38	1975	-798.97	-37.58	222.28	44%	7/23/79	-1178.98	1975	-805.93
	3 months	-38.52	219.60	47%	8/11/82	-1692.94	1975	-470.02	-45.53	218.00	47%	4/22/80	-946.94	1975	-592.04
	1 year	-62.57	226.68	59%	7/28/82	-1020.07	1982	-482.38	-96.71	263.48	59%	12/8/94	-1348.43	1995	-521.59
3-month US$ LIBOR	1 month	-0.54	223.85	31%	2/1/94	-1857.34	1994	-246.27	5.73	184.66	32%	4/8/94	-988.43	1994	-196.43
	3 months	2.30	222.76	28%	2/14/94	-1317.56	1994	-435.28	-20.47	261.85	29%	5/19/88	-1537.49	1988	-467.17
	1 year	-45.03	354.35	28%	12/17/93	-2003.32	1993	-672.09	-103.62	483.49	27%	3/10/88	-2453.58	1993	-721.40
Ten-Year Treasury Yield	1 month	-49.09	317.79	37%	9/21/79	-3671.87	1977	-311.83	-33.29	226.90	37%	3/27/90	-1135.83	1977	-315.71
	3 months	-60.01	326.09	38%	8/7/79	-2427.11	1979	-615.77	-53.65	283.77	38%	4/17/78	-1470.45	1977	-733.85
	1 year	-108.41	404.39	41%	2/20/79	-3060.70	1979	-1036.84	-70.75	309.34	41%	5/10/83	-1558.62	1977	-1097.21
Deutschemark Exchange Rate	1 month	2.78	212.29	33%	2/26/91	-1864.95	1991	-157.45	-1.16	197.85	33%	2/7/92	-1249.28	1983	-81.71
	3 months	2.22	226.19	33%	2/8/91	-1961.09	1991	-273.07	3.03	197.86	33%	10/12/93	-1465.33	1984	-122.18
	1 year	18.98	184.05	34%	8/21/92	-1051.50	1980	-457.95	29.61	148.80	34%	8/24/92	-773.64	1980	-315.95
Monthly Observations															
S&P 500 Stock Index	2 years	-50.53	200.28	77%	Sep-85	-870.96	1985	-508.86	-75.11	209.53	77%	Sep-85	-711.31	1984	-443.94
	5 years	-205.00	253.29	97%	Jul-82	-1213.16	1982	-762.88	-211.61	261.65	97%	Jul-82	-1192.75	1982	-752.26
3-mo. US$ LIBOR	2 years	-39.62	343.37	27%	Feb-93	-1250.74	1993	-1108.11	-54.70	366.25	27%	Feb-93	-1334.10	1993	-1190.01
	5 years	145.08	8.30	0%	Aug-90	128.18	1990	129.04	145.08	8.30	0%	Aug-90	128.18	1990	129.04
Ten-Year Treasury Yield	2 years	-101.57	443.87	34%	Sep-79	-2029.32	1979	-1309.51	-19.29	257.75	34%	Sep-77	-1100.61	1977	-901.74
	5 years	26.40	313.65	18%	Aug-77	-1606.78	1977	-902.55	91.66	157.25	18%	Aug-77	-874.68	1977	-428.76
Deutschemark Exchange Rate	2 years	47.24	134.81	32%	Jun-80	-400.20	1980	-287.60	48.42	131.37	32%	Jun-80	-399.13	1980	-275.17
	5 years	85.86	159.58	23%	Feb-80	-560.92	1980	-267.29	88.46	154.73	23%	Feb-80	-515.74	1980	-261.68

Table 2C

Return and Risk in Writing Options with Estimated Volatilities: At the Money Puts

The table reports the performance of a strategy of writing unhedged at the money put options each period for several maturities. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500	1 month	11.19	190.99	39%	9/25/87	-1985.80	1987	-94.89	24.37	131.09	39%	6/6/78	-651.68	1977	-71.25
Stock Index	3 months	23.94	178.72	32%	8/31/87	-1210.56	1987	-156.42	39.51	127.34	32%	6/26/81	-832.86	1981	-147.64
	1 year	36.14	210.65	24%	8/11/81	-1263.97	1981	-537.05	47.51	163.20	24%	3/18/81	-921.76	1981	-377.25
3-month	1 month	-14.38	173.77	50%	10/9/84	-1204.95	1991	-124.58	-11.12	145.23	50%	7/24/86	-578.42	1991	-116.47
US$ LIBOR	3 months	-30.39	196.33	54%	10/11/84	-1122.51	1991	-164.07	-25.38	161.60	54%	6/5/86	-766.12	1991	-183.70
	1 year	-85.82	244.83	61%	9/5/84	-921.15	1990	-641.83	-70.53	176.79	63%	9/12/85	-538.67	1991	-344.79
Ten-Year	1 month	-18.81	182.93	49%	5/10/79	-1264.93	1995	-116.77	-9.94	150.44	49%	2/7/86	-583.80	1995	-103.01
Treasury Yield	3 months	-24.86	190.39	46%	1/14/86	-879.41	1985	-210.06	-18.22	170.36	46%	1/14/86	-663.37	1985	-182.29
	1 year	-45.42	200.88	50%	4/11/85	-776.68	1985	-490.10	-35.98	179.66	50%	4/4/85	-600.35	1985	-357.81
Deutschemark	1 month	-31.95	199.28	53%	4/7/80	-1489.14	1977	-191.37	-32.56	172.04	53%	4/9/82	-854.72	1977	-124.82
Exchange Rate	3 months	-60.26	236.14	55%	9/28/77	-1725.11	1977	-482.25	-56.32	204.22	55%	9/12/77	-882.66	1985	-273.94
	1 year	-220.57	783.81	54%	6/28/77	-12535.31	1977	-2767.35	-117.82	275.20	54%	11/29/78	-1228.05	1985	-584.35
Monthly Observations															
S&P 500	2 years	108.81	50.13	8%	Jul-80	-443.22	1976	19.51	104.11	75.67	9%	Jul-80	-746.73	1976	-33.96
Stock Index	5 years	160.81	21.09	0%	Sep-90	127.98	1990	128.93	160.81	21.09	0%	Sep-90	127.98	1990	128.93
3-mo.	2 years	-58.94	212.47	64%	Sep-90	-601.90	1990	-507.21	-80.90	198.20	65%	Nov-89	-445.01	1990	-397.60
US$ LIBOR	5 years	-58.72	173.83	94%	Jan-89	-381.10	1988	-332.48	-90.50	164.78	94%	Sep-87	-394.66	1988	-322.00
Ten-Year	2 years	-22.48	174.96	56%	Jun-84	-553.65	1984	-440.56	-12.36	151.94	56%	Jun-84	-401.54	1984	-333.26
Treasury Yield	5 years	-50.93	199.46	71%	Aug-81	-552.25	1981	-432.81	-40.12	182.81	71%	Aug-81	-409.91	1981	-324.90
Deutschemark	2 years	-158.80	373.94	57%	Sep-77	-1180.70	1977	-889.15	-103.96	287.31	57%	Sep-76	-948.43	1976	-743.37
Exchange Rate	5 years	-197.47	416.68	73%	Dec-83	-1124.67	1985	-947.40	-153.84	347.29	73%	Feb-85	-853.79	1985	-763.20

Table 2D

Return and Risk in Writing Options with Estimated Volatilities: Out of the Money Puts

The table reports the performance of a strategy of writing unhedged out of the money put options each period for several maturities.The strike price is always set 0.4 standard deviations below the current spot. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500	1 month	4.60	286.52	25%	9/25/87	-3435.32	1987	-182.83	32.42	162.02	25%	6/6/78	-1191.12	1977	-77.61
Stock Index	3 months	16.98	258.13	19%	8/31/87	-2063.69	1987	-306.90	45.92	157.23	19%	6/26/81	-1342.56	1981	-171.88
	1 year	31.48	292.59	14%	8/11/81	-2196.90	1981	-812.89	51.34	203.81	14%	8/13/81	-1408.11	1981	-508.29
3-month	1 month	-22.10	248.51	37%	10/9/84	-1997.95	1986	-156.27	-12.08	194.02	38%	3/25/85	-1080.93	1991	-132.81
US$ LIBOR	3 months	-47.46	290.55	38%	10/11/84	-1863.51	1984	-278.33	-32.96	224.32	39%	6/5/86	-1313.68	1991	-251.80
	1 year	-132.72	377.38	51%	9/5/84	-1517.68	1990	-1029.66	-94.18	250.24	52%	9/12/85	-961.30	1991	-521.68
Ten-Year	1 month	-32.86	261.22	36%	5/10/79	-2094.01	1995	-145.62	-12.68	195.08	36%	6/4/93	-853.06	1995	-115.12
Treasury Yield	3 months	-48.59	269.83	36%	1/14/86	-1443.43	1985	-302.11	-34.46	229.58	36%	7/9/91	-1038.79	1985	-250.09
	1 year	-78.11	284.69	43%	4/11/85	-1273.67	1985	-767.20	-58.51	239.84	43%	4/4/85	-877.05	1985	-491.56
Deutschemark	1 month	-47.44	292.23	39%	4/7/80	-2585.40	1977	-283.35	-45.31	248.63	40%	4/9/82	-2194.74	1979	-176.37
Exchange Rate	3 months	-89.87	360.59	42%	9/28/77	-3054.76	1977	-754.20	-88.19	320.08	43%	6/11/79	-1964.14	1985	-446.98
	1 year	-401.24	1611.63	47%	6/28/77	-29455.29	1977	-5513.99	-190.93	435.61	47%	7/26/76	-2532.44	1976	-1175.19
Monthly Observations															
S&P 500	2 years	116.71	27.98	2%	Jul-80	-243.18	1976	91.93	113.27	55.67	2%	Jul-80	-566.06	1976	54.05
Stock Index	5 years	160.81	21.09	0%	Sep-90	127.98	1990	128.93	160.81	21.09	0%	Sep-90	127.98	1990	128.93
3-mo.	2 years	-92.08	325.72	46%	Sep-90	-992.08	1990	-820.16	-119.49	323.69	46%	Aug-84	-872.74	1989	-642.29
US$ LIBOR	5 years	-67.34	264.27	51%	Jan-89	-607.42	1988	-512.77	-88.55	279.87	51%	Mar-82	-649.90	1988	-489.78
Ten-Year	2 years	-31.62	242.84	45%	Jun-84	-890.04	1984	-686.13	-9.55	190.99	45%	Jun-84	-564.71	1984	-461.29
Treasury Yield	5 years	-58.05	272.27	61%	Aug-81	-876.66	1981	-654.87	-39.80	235.80	61%	Aug-81	-577.94	1981	-438.89
Deutschemark	2 years	-258.44	584.97	48%	Sep-77	-2018.78	1985	-1498.65	-166.31	454.91	48%	Sep-76	-1995.95	1976	-1458.42
Exchange Rate	5 years	-278.63	651.07	52%	Apr-85	-1918.41	1985	-1595.40	-191.99	493.60	52%	Feb-85	-1340.71	1985	-1180.88

Table 3A

Return and Risk in Writing and Delta Hedging Options with Estimated Volatilities: At the Money Calls

The table reports the performance of a strategy of writing and hedging at the money call options each period for several maturities. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500	1 month	-0.07	33.38	61%	10/16/87	-536.22	1987	-23.99	-3.18	12.41	61%	8/12/86	-80.93	1986	-5.07
Stock Index	3 months	1.01	24.74	68%	10/16/87	-329.05	1987	-23.72	-2.51	5.14	68%	6/12/86	-45.34	1980	-5.25
	1 year	3.26	42.40	76%	1/9/87	-290.53	1987	-99.22	-4.66	3.52	76%	10/27/86	-54.66	1980	-10.75
3-month	1 month	-13.71	99.11	43%	5/16/91	-521.86	1994	-95.86	-24.40	103.42	43%	1/17/95	-527.14	1994	-117.20
US$ LIBOR	3 months	-32.06	115.14	43%	4/30/84	-682.32	1994	-236.75	-49.87	124.99	43%	11/25/94	-543.48	1994	-274.21
	1 year	-108.34	149.64	38%	4/4/94	-810.39	1994	-403.44	-132.56	179.89	36%	4/11/94	-962.44	1994	-541.58
Ten-Year	1 month	-24.68	74.18	50%	9/20/79	-929.23	1978	-59.72	-26.56	56.26	50%	4/18/78	-342.08	1978	-67.87
Treasury Yield	3 months	-34.54	71.12	53%	7/23/79	-567.52	1978	-109.04	-41.05	64.17	53%	4/18/78	-331.79	1978	-104.49
	1 year	-59.11	92.25	50%	10/17/78	-553.46	1993	-244.71	-60.49	89.22	50%	1/12/94	-319.74	1977	-254.01
Deutschemark	1 month	3.80	36.25	47%	12/19/77	-372.73	1991	-18.84	-2.54	12.20	47%	2/3/93	-70.61	1988	-4.42
Exchange Rate	3 months	2.75	27.16	45%	3/6/95	-147.93	1991	-19.74	-1.67	4.99	45%	4/17/96	-42.83	1982	-5.01
	1 year	-0.43	24.21	46%	3/11/91	-96.95	1991	-31.47	-1.28	4.39	46%	8/10/88	-12.80	1980	-8.63
Monthly Observations															
S&P 500	2 years	-0.36	22.35	92%	Jun-86	-57.17	1986	-43.87	-6.71	6.81	91%	Oct-80	-21.37	1982	-14.24
Stock Index	5 years	-11.93	10.11	100%	Jun-86	-28.89	1986	-26.02	-12.38	5.59	100%	Feb-79	-21.72	1979	-21.05
3-mo.	2 years	-116.02	184.29	36%	Feb-93	-768.90	1993	-612.18	-158.12	168.47	35%	Feb-93	-793.10	1993	-631.91
US$ LIBOR	5 years	-75.44	71.07	5%	Aug-87	-251.02	1987	-174.99	-163.20	109.18	5%	Aug-86	-423.91	1986	-288.57
Ten-Year	2 years	-60.72	82.64	43%	Jan-87	-303.88	1993	-193.81	-55.86	83.03	43%	Jan-87	-320.92	1986	-187.08
Treasury Yield	5 years	-64.93	68.35	29%	Sep-79	-259.36	1979	-179.73	-52.37	69.16	29%	Aug-86	-306.56	1986	-218.41
Deutschemark	2 years	-2.72	25.39	43%	Nov-90	-109.04	1978	-33.13	-1.24	10.53	43%	Feb-89	-25.43	1980	-12.21
Exchange Rate	5 years	-2.94	23.79	27%	Aug-78	-40.02	1978	-31.11	-2.65	8.68	27%	Apr-81	-16.80	1979	-13.29

Table 3B

Return and Risk in Writing and Delta Hedging Options with Estimated Volatilities: Out of the Money Calls

The table reports the performance of a strategy of writing and hedging out of the money call options each period for several maturities. The strike price is always set 0.4 standard deviations above the current spot. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500	1 month	2.91	55.26	44%	10/20/87	-840.49	1987	-32.20	-3.19	21.95	44%	1/28/88	-195.82	1982	-7.27
Stock Index	3 months	3.93	41.75	47%	10/16/87	-397.63	1982	-32.46	-3.35	9.52	47%	1/14/88	-107.12	1980	-7.37
	1 year	14.37	65.00	59%	11/18/86	-425.52	1987	-101.79	-5.71	5.96	59%	10/20/86	-80.63	1980	-13.70
3-month	1 month	-15.26	134.43	31%	5/16/91	-891.65	1994	-125.76	-30.53	142.42	32%	8/23/82	-1110.87	1994	-146.74
US$ LIBOR	3 months	-40.16	161.92	28%	4/30/84	-1116.23	1994	-335.17	-68.35	189.73	29%	4/4/88	-1230.66	1994	-398.80
	1 year	-148.01	239.14	28%	2/21/94	-1321.29	1994	-624.51	-203.94	320.18	27%	4/11/94	-1710.38	1994	-970.06
Ten-Year	1 month	-36.52	117.86	37%	9/20/79	-1613.13	1987	-87.42	-40.18	84.35	37%	2/1/78	-554.81	1978	-105.61
Treasury Yield	3 months	-49.15	111.83	38%	7/23/79	-955.31	1978	-160.87	-59.21	99.22	38%	3/29/78	-695.25	1978	-172.80
	1 year	-81.08	141.11	41%	10/17/78	-911.68	1993	-354.67	-86.02	140.35	41%	9/2/77	-732.73	1977	-521.70
Deutschemark	1 month	5.26	60.88	33%	12/19/77	-526.06	1991	-43.14	-3.15	22.87	33%	7/23/91	-125.34	1996	-6.90
Exchange Rate	3 months	6.25	45.53	33%	3/7/95	-223.81	1991	-41.43	-1.89	8.95	33%	5/20/91	-58.96	1980	-6.20
	1 year	0.24	43.80	34%	2/25/91	-202.31	1991	-61.66	-1.60	6.19	34%	4/27/90	-42.32	1980	-11.44
Monthly Observations															
S&P 500	2 years	7.82	47.30	77%	May-86	-109.74	1986	-87.20	-7.20	14.77	77%	Dec-89	-58.79	1982	-21.34
Stock Index	5 years	-13.47	23.42	97%	Jun-86	-49.87	1986	-46.27	-16.14	8.89	97%	Aug-77	-40.16	1979	-27.93
3-mo.	2 years	-145.79	278.84	27%	Feb-93	-1213.66	1993	-971.22	-219.28	264.66	27%	Feb-93	-1282.16	1993	-1031.12
US$ LIBOR	5 years	-65.47	87.07	0%	Aug-87	-294.19	1987	-195.19	-193.18	160.95	0%	Aug-86	-755.13	1986	-448.78
Ten-Year	2 years	-83.62	123.41	34%	Jan-87	-440.34	1979	-300.95	-71.74	116.47	34%	Jan-87	-487.17	1977	-299.59
Treasury Yield	5 years	-80.69	99.37	18%	Sep-79	-437.31	1979	-300.37	-56.61	96.47	18%	Nov-86	-408.03	1986	-300.15
Deutschemark	2 years	0.59	36.82	32%	Sep-78	-77.77	1979	-44.83	-1.24	17.77	32%	Sep-76	-49.68	1988	-20.23
Exchange Rate	5 years	-3.62	28.16	23%	Sep-78	-58.44	1978	-40.02	-3.37	10.13	23%	Dec-78	-20.34	1979	-19.05

Table 3C

Return and Risk in Writing and Delta Hedging Options with Estimated Volatilities: At the Money Puts

The table reports the performance of a strategy of writing and hedging at the money put options each period for several maturities. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500	1 month	1.80	41.59	39%	10/16/87	-602.84	1987	-26.54	-0.76	15.38	39%	8/12/86	-88.78	1986	-4.11
Stock Index	3 months	5.00	34.72	32%	10/16/87	-451.05	1987	-30.87	2.23	7.73	32%	6/12/86	-53.67	1995	-1.42
	1 year	8.70	73.32	24%	1/9/87	-537.63	1987	-172.28	5.42	10.18	24%	10/27/86	-67.89	1987	-2.32
3-month	1 month	25.95	106.86	50%	11/12/90	-540.44	1989	-33.70	27.26	107.77	50%	12/18/89	-518.37	1989	-37.09
US$ LIBOR	3 months	30.42	118.75	54%	12/19/95	-687.48	1989	-87.83	31.30	113.41	54%	12/14/89	-487.20	1989	-112.62
	1 year	60.19	116.25	61%	3/5/90	-388.62	1979	-131.01	60.04	125.21	63%	6/1/95	-276.08	1989	-164.03
Ten-Year	1 month	3.51	58.71	49%	10/19/81	-390.54	1981	-28.46	1.87	47.93	49%	9/13/82	-227.31	1982	-33.16
Treasury Yield	3 months	10.04	57.24	46%	10/22/79	-383.25	1982	-33.87	7.07	46.39	46%	8/18/81	-182.56	1982	-47.76
	1 year	30.99	53.25	50%	10/22/79	-255.12	1979	-33.31	28.18	47.36	50%	10/6/81	-141.89	1981	-51.17
Deutschemark	1 month	5.41	47.78	53%	12/19/77	-667.40	1977	-18.81	-1.10	14.58	53%	7/20/87	-71.47	1991	-4.65
Exchange Rate	3 months	6.31	38.49	55%	12/14/77	-368.30	1977	-23.55	1.12	7.12	55%	4/17/96	-61.99	1992	-2.72
	1 year	-6.26	99.27	54%	6/28/77	-1928.69	1977	-268.01	6.50	11.37	54%	10/31/78	-16.88	1991	-2.65
Monthly Observations															
S&P 500	2 years	14.22	39.23	8%	Jun-86	-101.59	1986	-75.34	9.30	23.33	9%	Oct-80	-47.32	1984	-10.75
Stock Index	5 years	11.49	35.15	0%	Jun-86	-72.94	1986	-59.42	13.57	13.60	0%	Apr-80	-6.07	1990	-0.70
3-mo.	2 years	136.15	118.36	64%	Jan-89	-45.23	1989	18.24	123.96	112.47	65%	Nov-88	-99.55	1989	-16.42
US$ LIBOR	5 years	252.82	118.19	94%	Jan-87	73.90	1986	114.16	272.20	105.00	94%	Jan-87	43.37	1986	97.21
Ten-Year	2 years	48.70	61.23	56%	Jan-81	-130.54	1979	-63.62	49.89	44.74	56%	Dec-80	-53.77	1989	-4.08
Treasury Yield	5 years	114.42	86.36	71%	Sep-79	-78.99	1979	-63.50	119.77	61.92	71%	Nov-78	26.93	1979	33.35
Deutschemark	2 years	-4.70	49.12	57%	Aug-78	-271.28	1978	-137.30	5.81	18.04	57%	Sep-76	-70.46	1976	-24.17
Exchange Rate	5 years	-17.21	70.02	73%	Aug-78	-371.67	1978	-188.05	17.69	26.53	73%	Apr-81	-37.02	1981	-6.99

Table 3D

Return and Risk in Writing and Delta Hedging Options with Estimated Volatilities: Out of the Money Puts

The table reports the performance of a strategy of writing and hedging out of the money put options each period for several maturities. The strike price is always set 0.4 standard deviations below the current spot. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. The period is daily in the upper panel and monthly in the lower. Option prices are computed by using as the volatility input either the minimum RMSE volatility estimate using historical data (left panel) or the realized volatility over the option's lifetime (right panel). The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

		Minimum RMSE Volatility Forecast using Historical Data							Realized Volatility						
Underlying	Maturity	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean	Mean Return	Standard Deviation	% In-the-money	Worst Case Date	Worst Case Return	Worst Full Year Year	Worst Full Year Mean
Daily Observations															
S&P 500 Stock Index	1 month	-2.24	69.47	25%	10/16/87	-1332.87	1987	-56.06	-2.13	27.17	25%	9/19/84	-129.80	1986	-9.54
	3 months	-0.23	56.25	19%	10/16/87	-1025.80	1987	-70.77	2.19	12.46	19%	5/26/81	-96.57	1995	-2.25
	1 year	-4.14	131.55	14%	1/16/87	-889.80	1987	-411.19	6.10	13.51	14%	11/11/76	-50.99	1984	-2.06
3-month US$ LIBOR	1 month	29.97	148.64	37%	11/12/90	-815.13	1986	-48.36	36.28	157.34	38%	6/27/86	-883.66	1989	-41.73
	3 months	34.30	166.67	38%	12/19/95	-1116.09	1989	-121.40	36.97	154.25	39%	11/7/89	-689.39	1989	-148.47
	1 year	74.64	171.06	51%	3/5/90	-697.83	1990	-234.99	66.46	203.76	52%	3/29/95	-607.70	1989	-330.86
Ten-Year Treasury Yield	1 month	0.93	87.93	36%	10/19/81	-618.02	1981	-42.73	-1.62	70.26	36%	12/28/78	-380.25	1982	-42.16
	3 months	7.07	82.33	36%	10/23/79	-509.15	1980	-55.13	2.50	62.01	36%	9/3/91	-255.34	1982	-73.76
	1 year	34.31	75.06	43%	10/22/79	-372.82	1979	-46.08	29.37	66.37	43%	1/15/82	-242.53	1982	-75.62
Deutschemark Exchange Rate	1 month	4.60	84.57	39%	12/19/77	-1237.95	1977	-55.78	-2.16	30.30	40%	6/28/79	-242.85	1992	-8.04
	3 months	4.61	71.17	42%	12/7/77	-767.65	1977	-66.91	1.24	15.86	43%	6/15/79	-126.93	1992	-5.19
	1 year	-25.49	247.54	47%	6/28/77	-5176.43	1977	-694.30	9.47	21.88	47%	12/21/76	-77.42	1991	-2.76
Monthly Observations															
S&P 500 Stock Index	2 years	4.69	55.63	2%	Aug-80	-263.99	1986	-91.88	2.61	39.34	2%	Aug-80	-201.50	1983	-51.19
	5 years	8.05	45.54	0%	Jun-86	-106.20	1986	-80.75	10.17	17.49	0%	Apr-80	-29.73	1980	-13.33
3-mo. US$ LIBOR	2 years	196.77	181.17	46%	Jan-89	-80.19	1993	29.65	178.98	183.05	46%	Feb-89	-205.95	1989	-32.28
	5 years	373.65	243.70	51%	Jan-87	-27.91	1986	114.26	435.46	223.46	51%	Jan-87	-37.10	1986	97.88
Ten-Year Treasury Yield	2 years	67.28	84.36	45%	Jan-81	-187.99	1979	-70.52	68.30	61.79	45%	Dec-80	-70.05	1980	1.53
	5 years	161.56	129.18	61%	Sep-79	-114.84	1979	-88.29	165.97	101.93	61%	Nov-78	24.53	1979	30.05
Deutschemark Exchange Rate	2 years	-26.37	126.30	48%	Apr-78	-806.06	1978	-404.27	4.02	31.94	48%	Jul-76	-143.12	1976	-58.31
	5 years	-59.86	124.85	52%	Aug-78	-671.33	1978	-353.34	21.38	44.66	52%	Jul-81	-186.28	1981	-22.23

Table 4A
Return and Risk in Writing Call Options with Alternative Volatility Estimates and Different Degrees of Moneyness

The table reports the performance of a strategy of writing and hedging call options each day for 1 month options, or each month for 2 year options. For Deep out of the Money, Out of the Money, At the Money, and In the Money contracts, the strike prices are set, respectively, +1.0, +0.4, 0.0, and -0.4 standard deviations above or below the current spot price. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. Alternative option values and hedge ratios are computed using the realized volatility, the minimum RMSE forecast, or a volatility estimate from a suboptimal historical data sample. For exponentially weighted estimates, the mean lag in days is shown in parentheses. The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

Underlying	Maturity	Volatility Estimate	Deep out of the Money Mean	Deep out of the Money Std. Dev.	Deep out of the Money Worst Year	Out of the Money Mean	Out of the Money Std. Dev.	Out of the Money Worst Year	At the Money Mean	At the Money Std. Dev.	At the Money Worst Year	In the Money Mean	In the Money Std. Dev.	In the Money Worst Year
S&P 500 Stock Index	1 Month	Realized	8.81	75.79	-10.78	-3.19	21.95	-7.27	-3.18	12.41	-5.07	-2.00	7.60	-4.17
		Exp. wgt (47)	8.48	113.92	-71.53	2.91	55.26	-32.20	-0.07	33.38	-23.99	-1.64	20.40	-18.24
		5 years daily	22.88	176.82	-192.21	17.56	56.27	-57.92	11.78	34.21	-32.65	7.18	21.92	-20.47
	2 Years	Realized	-2.78	51.41	-34.97	-7.20	14.77	-21.34	-6.71	6.81	-14.24	-5.27	4.42	-9.82
		All available	7.51	120.97	-330.32	7.82	47.30	-87.20	-0.36	22.35	-43.87	-3.22	10.80	-20.59
		3 months daily	-90.50	254.15	-816.31	-23.13	58.12	-106.65	-14.30	27.90	-56.17	-9.34	14.67	-26.06
3 month US$ LIBOR	1 Month	Realized	-35.72	278.25	-223.13	-30.53	142.42	-146.74	-24.40	103.42	-117.20	-18.60	77.69	-87.49
		Exp. wgt (37)	-28.25	255.65	-218.52	-15.26	134.43	-125.76	-13.71	99.11	-95.86	-14.44	75.80	-74.92
		5 years daily	3.09	266.34	-225.07	12.74	122.98	-127.36	8.99	89.30	-97.46	3.49	68.77	-76.19
	2 Years	Realized	-388.36	636.53	-2524.14	-219.28	264.66	-1031.12	-158.12	168.47	-631.91	-120.50	119.64	-427.06
		60 months	-240.75	588.81	-2232.65	-145.79	278.84	-971.22	-116.02	184.29	-612.18	-97.74	131.18	-419.01
		3 months daily	-1082.95	3696.30	-9214.60	-293.49	492.43	-1697.72	-181.09	222.24	-822.71	-129.48	135.39	-486.96
Ten-Year Treasury Yield	1 Month	Realized	-58.87	195.14	-235.12	-40.18	84.35	-105.61	-26.56	56.26	-67.87	-17.42	40.90	-46.05
		3 months daily	-74.54	283.18	-230.63	-36.52	117.86	-87.42	-24.68	74.18	-59.72	-17.58	50.46	-46.69
		5 years daily	-226.52	1210.96	-3560.60	-46.85	170.09	-438.89	-22.13	80.93	-141.50	-12.64	50.37	-54.98
	2 Years	Realized	-97.93	187.87	-602.18	-71.74	116.47	-299.59	-55.86	83.03	-187.08	-42.91	61.24	-146.38
		All available	-161.00	296.65	-908.10	-83.62	123.41	-300.95	-60.72	82.64	-193.81	-46.67	59.84	-149.48
		3 months daily	-1549.76	5428.99	-11648.51	-200.30	381.52	-1054.93	-95.89	132.91	-332.96	-59.50	70.43	-171.98
Deutschemark Exchange Rate	1 Month	Realized	2.18	90.90	-17.51	-3.15	22.87	-6.90	-2.54	12.20	-4.42	-1.38	7.55	-4.29
		12 months daily	-3.56	136.59	-125.48	5.26	60.88	-43.14	3.80	36.25	-18.84	1.17	22.26	-11.04
		5 years daily	-11.20	172.55	-265.69	6.32	65.20	-69.39	6.34	36.91	-31.33	4.07	23.15	-15.87
	2 Years	Realized	-3.26	25.89	-32.37	-1.24	17.77	-20.23	-1.24	10.53	-12.21	-1.27	7.55	-8.31
		All available	-2.58	67.94	-81.04	0.59	36.82	-44.83	-2.72	25.39	-33.13	-4.71	15.66	-29.62
		3 months daily	-67.20	277.09	-484.03	-10.16	83.47	-160.55	-6.34	45.45	-67.63	-6.20	28.84	-69.11

Table 4B
Return and Risk in Writing Put Options with Alternative Volatility Estimates and Different Degrees of Moneyness

The table reports the performance of a strategy of writing and hedging put options each day for 1 month options, or each month for 2 year options. For Deep out of the Money, Out of the Money, At the Money, and In the Money contracts, the strike prices are set, respectively, -1.0, -0.4, 0.0, and +0.4 standard deviations below or above the current spot price. S&P 500 and Deutschemark options are hedged with the underlying asset; the interest rate options are hedged using futures, as described in the text. Alternative option values and hedge ratios are computed using the realized volatility, the minimum RMSE forecast, or a volatility estimate from a suboptimal historical data sample. For exponentially weighted estimates, the mean lag in days is shown in parentheses. The hedge is rebalanced to be delta neutral every period, using the current volatility estimate. The strategy always writes enough options to produce $100 of option premium.

Underlying	Maturity	Volatility	Deep out of the Money			Out of the Money			At the Money			In the Money		
		Estimate	Mean	Std. Dev.	Worst Year	Mean	Std. Dev.	Worst Year	Mean	Std. Dev.	Worst Year	Mean	Std. Dev.	Worst Year
S&P 500 Stock Index	1 Month	Realized	-1.93	63.39	-20.04	-2.13	27.17	-9.54	-0.76	15.38	-4.11	0.73	9.56	-1.35
		Exp. wgt (47)	-23.33	170.23	-181.91	-2.24	69.47	-56.06	1.80	41.59	-26.54	2.67	25.24	-12.71
		5 years daily	-6.74	231.68	-340.40	13.92	70.70	-91.61	14.63	40.68	-43.94	12.33	24.73	-20.63
	2 Years	Realized	-5.03	68.08	-162.95	2.61	39.34	-51.19	9.30	23.33	-10.75	9.43	15.12	-2.69
		All available	-9.91	93.13	-183.17	4.69	55.63	-91.88	14.22	39.23	-75.34	15.62	29.38	-56.11
		3 months daily	-896.66	6052.10	-10952.68	-166.12	496.74	-1220.67	-51.51	145.15	-403.05	-13.43	61.01	-155.18
3 month US$ LIBOR	1 Month	Realized	67.87	366.22	-81.16	36.28	157.34	-41.73	27.26	107.77	-37.09	20.82	77.71	-27.00
		Exp. wgt (37)	19.03	268.85	-143.51	29.97	148.64	-48.36	25.95	106.86	-33.70	19.21	79.25	-24.93
		5 years daily	49.11	298.10	-101.54	52.04	127.38	-24.23	43.32	88.81	-8.08	32.66	66.54	-3.19
	2 Years	Realized	294.55	401.37	3.32	178.98	183.05	-32.28	123.96	112.47	-16.42	87.52	76.59	-4.66
		60 months	337.69	378.13	63.56	196.77	181.17	29.65	136.15	118.36	18.24	96.62	83.14	11.61
		3 months daily	625.05	1914.42	-507.85	197.64	255.80	-137.08	132.72	136.61	-18.28	95.08	90.76	4.64
Ten-Year Treasury Yield	1 Month	Realized	-6.32	143.43	-73.00	-1.62	70.26	-42.16	1.87	47.93	-33.16	4.51	36.10	-21.25
		3 months daily	-14.96	181.82	-113.34	0.93	87.93	-42.73	3.51	58.71	-28.46	4.41	41.58	-20.77
		5 years daily	-78.57	1010.81	-1458.34	-3.81	122.61	-194.30	4.36	66.53	-85.41	6.88	43.81	-42.90
	2 Years	Realized	106.10	107.68	11.51	68.30	61.79	1.53	49.89	44.74	-4.08	40.50	34.41	-5.92
		All available	100.13	133.05	-78.67	67.28	84.36	-70.52	48.70	61.23	-63.62	36.72	46.90	-54.83
		3 months daily	105.59	1012.07	-1397.03	58.22	141.66	-273.85	44.86	77.94	-139.18	36.65	51.79	-80.53
Deutschemark Exchange Rate	1 Month	Realized	6.66	119.99	-15.56	-2.16	30.30	-8.04	-1.10	14.58	-4.65	0.03	8.86	-3.41
		12 months daily	-15.78	248.61	-279.24	4.60	84.57	-55.78	5.41	47.78	-18.81	3.09	28.13	-14.05
		5 years daily	-13.26	246.22	-252.03	8.28	77.58	-71.15	8.73	44.82	-38.16	5.69	28.13	-23.97
	2 Years	Realized	19.23	113.34	-47.94	4.02	31.94	-58.31	5.81	18.04	-24.17	5.22	11.27	-8.13
		All available	-118.91	327.25	-844.20	-26.37	126.30	-404.27	-4.70	49.12	-137.30	-0.03	25.54	-50.81
		3 months daily	-15454.70	90416.03	-301145.3	-439.76	1616.61	-5835.24	-94.40	294.45	-835.89	-28.64	90.58	-271.65

Table 5A
Return and Risk in Writing and Delta Hedging Call Options with a Volatility Markup

The table reports the performance of a strategy of writing and hedging call options each day for 3 month options, or each month for 2 year options. Option prices are set by valuing the option with the appropriate model but with a volatility input that is "marked up" by 0%, 10%, 25%, or 50% (i.e., the best volatility forecast is multiplied by 1.0, 1.1, 1.25, or 1.5.) Hedging and hedge rebalancing are done with the unadjusted volatility. The table shows the mean return per $100 of option premium, the standard deviation of the return, the percentage of positions that lose money, and the average loss for losing positions. See also the notes to Table 4A.

			Deep Out of the Money				Out of the Money				At the Money				In the Money			
		Markup	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss
S&P 500	3 month	0%	3.7	87.6	41.1%	-71.5	3.9	41.8	45.2%	-31.9	1.0	24.7	46.8%	-18.9	-1.8	14.3	50.9%	-11.6
		10%	30.5	88.2	27.7%	-73.7	17.9	42.2	31.4%	-29.3	9.5	25.1	32.2%	-17.5	3.1	14.5	33.7%	-11.6
		25%	73.6	89.4	15.4%	-77.7	38.9	43.1	16.4%	-27.3	22.3	25.6	16.0%	-17.5	10.6	14.8	17.8%	-12.0
		50%	150.8	92.6	5.4%	-84.7	74.2	44.7	4.8%	-25.7	43.6	26.6	5.7%	-15.6	23.5	15.4	6.5%	-11.6
	2 year	0%	7.5	121.0	37.3%	-100.7	7.8	47.3	46.7%	-28.4	-0.4	22.3	59.9%	-14.1	-3.2	10.8	73.1%	-8.4
		10%	32.4	122.2	27.8%	-107.6	20.3	48.7	34.0%	-25.6	7.1	23.4	39.2%	-13.7	0.9	11.5	45.8%	-8.6
		25%	71.4	124.6	16.0%	-139.4	39.1	50.9	17.5%	-24.6	18.3	24.9	23.6%	-10.3	7.4	12.7	30.2%	-6.5
		50%	139.3	130.7	9.9%	-129.9	70.5	55.0	6.1%	-22.0	37.3	27.7	5.7%	-6.5	18.8	14.5	5.2%	-2.8
US $ LIBOR	3 month	0%	-70.3	308.9	47.1%	-244.5	-40.2	161.9	52.6%	-138.3	-32.1	115.1	57.3%	-100.1	-27.9	86.5	60.5%	-79.0
		10%	-39.4	308.9	35.2%	-291.5	-23.8	161.9	45.7%	-141.5	-21.9	115.1	51.5%	-100.7	-21.9	86.5	57.3%	-77.3
		25%	11.4	308.9	23.8%	-370.0	1.1	161.9	35.3%	-155.1	-6.6	115.1	43.7%	-102.0	-12.8	86.5	52.5%	-74.9
		50%	104.6	308.8	16.3%	-430.4	43.3	161.9	24.1%	-176.5	18.7	115.1	32.7%	-107.0	2.6	86.5	44.1%	-72.1
	2 year	0%	-233.7	543.1	67.4%	-367.4	-142.5	253.0	70.3%	-212.5	-113.5	166.5	80.4%	-145.8	-96.0	117.9	87.7%	-112.7
		10%	-198.7	543.8	54.3%	-417.2	-124.0	253.4	61.6%	-223.0	-102.1	166.8	71.0%	-153.2	-89.3	118.0	83.3%	-111.7
		25%	-141.7	544.8	35.5%	-568.8	-96.1	254.0	51.4%	-236.6	-85.0	167.1	60.9%	-160.7	-79.2	118.2	77.5%	-109.5
		50%	-37.9	546.4	23.2%	-744.5	-49.3	254.8	36.2%	-281.5	-56.9	167.6	49.3%	-167.5	-62.2	118.5	63.8%	-114.4
10 Year Treasury Yield	3 month	0%	-100.9	265.8	60.0%	-199.1	-49.2	111.8	63.3%	-95.7	-34.5	71.1	67.7%	-63.0	-25.9	49.2	69.5%	-46.0
		10%	-69.8	265.8	41.1%	-253.4	-32.7	111.8	49.1%	-104.8	-24.3	71.1	55.7%	-65.3	-19.9	49.2	62.3%	-45.0
		25%	-18.8	265.8	28.9%	-301.9	-7.7	111.9	34.8%	-118.5	-9.0	71.2	42.3%	-68.3	-10.7	49.2	52.2%	-43.6
		50%	74.9	265.8	18.2%	-361.2	34.7	111.9	23.4%	-124.8	16.5	71.2	27.4%	-74.0	4.8	49.2	35.1%	-46.2
	2 year	0%	-161.0	296.6	65.8%	-259.1	-83.6	123.4	67.4%	-134.8	-60.7	82.6	68.9%	-97.0	-46.7	59.8	74.1%	-69.3
		10%	-124.6	295.8	50.3%	-296.8	-64.4	123.3	57.0%	-138.2	-48.8	82.7	62.7%	-94.2	-39.6	59.9	67.4%	-68.9
		25%	-64.8	294.4	37.3%	-329.8	-35.1	123.1	46.1%	-137.8	-30.8	82.8	52.9%	-92.5	-29.0	60.1	59.1%	-67.2
		50%	44.6	291.9	26.4%	-340.0	14.4	122.9	32.1%	-137.8	-1.1	83.0	40.4%	-86.3	-10.9	60.4	50.8%	-58.9
Deutschemark	3 month	0%	2.4	95.6	39.0%	-79.6	6.2	45.5	38.8%	-35.8	2.7	27.2	41.3%	-21.5	0.4	16.8	45.0%	-13.0
		10%	30.6	96.0	24.5%	-91.7	21.0	45.7	24.7%	-38.0	11.8	27.3	27.2%	-21.9	5.6	17.0	29.4%	-13.7
		25%	76.2	97.3	14.3%	-99.4	43.3	46.2	13.4%	-39.0	25.4	27.7	14.0%	-23.7	13.7	17.3	15.6%	-15.1
		50%	159.1	101.8	7.6%	-76.8	81.0	48.0	5.3%	-35.0	48.1	28.9	5.5%	-22.9	27.4	18.2	5.8%	-17.7
	2 year	0%	-2.6	67.9	59.0%	-40.8	0.6	36.8	57.5%	-22.1	-2.7	25.4	56.1%	-18.0	-4.7	15.7	64.6%	-12.4
		10%	26.8	73.0	34.4%	-35.0	15.8	40.8	35.8%	-19.8	6.6	27.0	42.5%	-15.2	0.7	16.3	50.5%	-10.6
		25%	75.1	86.6	13.7%	-31.5	39.2	48.6	17.5%	-15.3	20.8	30.6	18.9%	-15.5	9.1	18.1	28.3%	-9.8
		50%	164.0	125.8	1.4%	-7.4	79.3	65.6	2.4%	-13.7	44.8	38.8	5.7%	-11.5	23.5	22.5	6.6%	-16.3

Table 5B
Return and Risk in Writing and Delta Hedging Put Options with a Volatility Markup

The table reports the performance of a strategy of writing and hedging put options each day for 3 month options, or each month for 2 year options. Option prices are set by valuing the option with the appropriate model but with a volatility input that is "marked up" by 0%, 10%, 25%, or 50% (i.e., the best volatility forecast is multiplied by 1.0, 1.1, 1.25, or 1.5.) Hedging and hedge rebalancing are done with the unadjusted volatility. The table shows the mean return per $100 of option premium, the standard deviation of the return, the percentage of positions that lose money, and the average loss for losing positions. See also the notes to Table 4A.

			Deep Out of the Money				Out of the Money				At the Money				In the Money			
		Markup	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss	Mean	Std Dev	Losses	Avg Loss
S&P 500	3 month	0%	-31.3	166.0	43.6%	-125.1	-0.2	56.3	41.8%	-46.5	5.0	34.7	39.6%	-27.3	4.3	21.4	37.6%	-17.2
		10%	4.8	165.6	25.2%	-167.6	19.2	55.9	25.3%	-51.0	17.3	34.5	24.4%	-27.9	11.7	21.2	25.0%	-16.5
		25%	66.0	165.1	15.3%	-197.6	49.3	55.5	13.3%	-54.0	35.9	34.2	12.3%	-28.0	22.9	21.1	13.3%	-14.7
		50%	182.2	165.5	7.5%	-221.0	101.0	55.6	4.7%	-50.9	66.9	34.3	3.8%	-26.2	41.6	21.1	2.7%	-19.8
	2 year	0%	-9.9	93.1	35.4%	-102.6	4.7	55.6	33.0%	-56.9	14.2	39.2	26.4%	-35.9	15.6	29.4	23.6%	-24.7
		10%	43.5	88.6	18.9%	-109.0	34.4	53.2	20.3%	-48.6	33.5	39.4	13.7%	-40.8	27.6	29.9	14.2%	-27.0
		25%	140.5	87.9	7.5%	-83.4	82.0	52.8	9.4%	-41.5	63.0	42.0	9.0%	-30.2	45.7	31.8	9.0%	-23.8
		50%	339.1	132.7	0.5%	-5.3	167.2	66.3	0.5%	-7.1	113.3	52.1	2.4%	-11.0	75.9	37.6	4.2%	-14.3
US $ LIBOR	3 month	0%	21.2	317.4	28.5%	-302.1	34.3	166.7	27.6%	-162.5	30.4	118.8	28.9%	-110.5	23.9	86.9	29.4%	-80.2
		10%	52.2	317.4	22.0%	-356.4	50.7	166.7	23.9%	-169.6	40.6	118.8	26.4%	-110.5	29.9	86.9	28.0%	-78.1
		25%	102.9	317.4	17.5%	-392.5	75.6	166.7	19.4%	-181.0	55.8	118.7	22.4%	-113.6	39.0	86.9	25.4%	-76.5
		50%	196.2	317.3	13.3%	-411.0	117.7	166.7	15.6%	-178.9	81.2	118.7	17.4%	-117.7	54.4	86.9	20.7%	-77.1
	2 year	0%	378.3	441.4	6.5%	-24.7	214.2	199.9	5.1%	-17.9	145.7	126.5	3.6%	-9.3	102.1	86.2	1.4%	-14.2
		10%	413.2	440.5	2.2%	-9.6	232.6	199.5	2.9%	-3.3	157.1	126.3	1.4%	-6.3	108.8	86.0	1.4%	-7.6
		25%	470.2	439.1	0.0%	0.0	260.5	198.8	0.0%	0.0	174.2	125.9	0.0%	0.0	119.0	85.8	0.0%	0.0
		50%	574.0	436.8	0.0%	0.0	307.3	197.7	0.0%	0.0	202.3	125.2	0.0%	0.0	136.0	85.4	0.0%	0.0
10 Year Treasury Yield	3 month	0%	-6.9	163.9	34.5%	-150.5	7.1	82.3	33.1%	-76.0	10.0	57.2	32.2%	-50.9	10.0	42.0	31.5%	-36.4
		10%	24.2	163.9	23.8%	-181.2	23.5	82.3	24.4%	-83.9	20.3	57.2	25.0%	-54.2	16.0	42.0	25.6%	-38.1
		25%	75.2	163.8	16.6%	-200.1	48.5	82.3	17.8%	-85.6	35.6	57.2	18.2%	-56.5	25.2	42.0	19.3%	-39.9
		50%	168.9	163.7	10.2%	-203.6	90.9	82.2	10.4%	-89.7	61.1	57.2	11.3%	-58.1	40.6	41.9	13.4%	-38.8
	2 year	0%	100.1	133.1	13.5%	-91.6	67.3	84.4	16.1%	-76.5	48.7	61.2	16.1%	-59.6	36.7	46.9	17.1%	-44.0
		10%	136.5	131.4	7.3%	-112.2	86.5	83.3	13.5%	-68.2	60.7	60.6	14.5%	-51.8	43.8	46.5	15.5%	-40.3
		25%	196.3	128.7	4.7%	-89.3	115.8	81.7	9.8%	-56.8	78.6	59.6	12.4%	-38.5	54.4	45.9	14.5%	-30.7
		50%	305.7	124.0	1.0%	-29.4	165.3	79.0	3.6%	-45.3	108.4	57.9	5.7%	-30.1	72.5	45.0	9.8%	-18.6
Deutschemark	3 month	0%	-25.1	224.1	39.8%	-159.9	4.6	71.2	37.6%	-52.9	6.3	38.5	35.0%	-30.0	4.9	22.7	33.1%	-17.8
		10%	9.7	223.0	25.3%	-207.1	23.3	70.9	23.6%	-60.4	18.1	38.5	23.1%	-30.7	12.0	22.7	21.8%	-18.2
		25%	68.4	221.0	16.8%	-240.1	52.2	70.8	13.6%	-66.5	35.9	38.6	11.6%	-35.9	22.7	23.0	11.0%	-20.5
		50%	179.6	218.3	8.5%	-298.6	101.8	71.5	5.3%	-87.9	65.6	39.6	5.1%	-37.6	40.7	23.7	4.5%	-20.5
	2 year	0%	-118.9	327.3	60.4%	-239.0	-26.4	126.3	54.7%	-80.4	-4.7	49.1	47.6%	-37.0	0.0	25.5	48.6%	-18.4
		10%	-69.7	322.0	41.0%	-297.6	0.8	123.4	35.4%	-90.9	12.9	47.7	27.8%	-38.0	10.8	25.1	29.2%	-15.4
		25%	18.8	312.5	23.1%	-412.0	44.2	120.0	13.7%	-157.1	39.7	47.4	9.9%	-52.0	27.2	25.9	5.7%	-17.8
		50%	197.9	303.0	13.7%	-379.0	121.5	119.6	7.5%	-162.4	85.4	52.4	3.8%	-49.4	54.7	30.8	0.9%	-19.9