CS 314

Problem Set 2

Sample Solution

1 Problem — Context-Free Languages

Are the following languages context-free or not? If yes, specify a context-free grammar in BNF notation that generates the language. If not, give an **informal** argument.

All of these languages are context free. Sample sets of rules are given, but other rules may also work.

1. { $a^n b^m c^o$ | m > n $\geq$ 0, o > 0} , with alphabet Σ = {a, b, c}
   $< S >::= < A >< B >< C >$
   $< A >::= a < A > b \mid \epsilon$
   $< B >::= b < B > \mid b$
   $< C >::= c < C > \mid c$
2. { $a^n b^{2n}$ | n $\geq$ 0 }, with alphabet Σ = {a, b}
   $< S >::= a < S > bb \mid \epsilon$
3. { ww^R | w $\in \Sigma^*$ and w^R is w in reverse }, with alphabet Σ ={a, b}
   $< S >::= a < S > a \mid b < S > b \mid \epsilon$
4. { $a^n b^m c^m d^n$ | n $\geq$ 0, m $\geq$ 0 }, with alphabet Σ = {a, b, c, d}
   $< S >::= a < S > d \mid < A > \mid \epsilon$
   $< A >::= b < A > c \mid \epsilon$
5. { w | w has no more than 5 symbols}, with alphabet Σ={a, b}
   $< S >::= < A >< A >< A >< A >< A >$
   $< A >::= a \mid b \mid \epsilon$

2 Problem — Derivation, Parse Tree, Ambiguity, Precedence & Associativity

A language that is a subset of the language of propositional logic may be defined as follows:

<start> ::= <expr>

<expr> :: = <expr> $\vee$ <expr> |
<expr> $\wedge$ <expr> |
<expr> $\rightarrow$ <expr> |
<const> | <var>

<const> :: = true | false

<var> :: = a | b | c | . . . | z

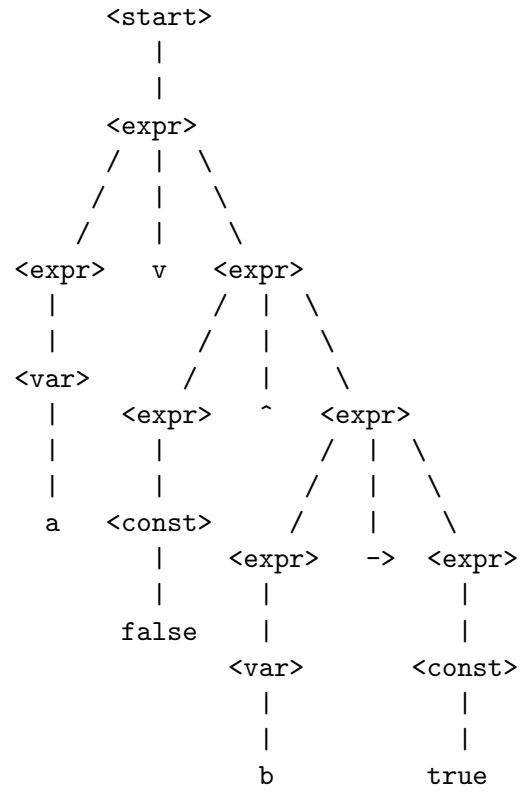
1. Give a leftmost and a rightmost derivation for the sentence

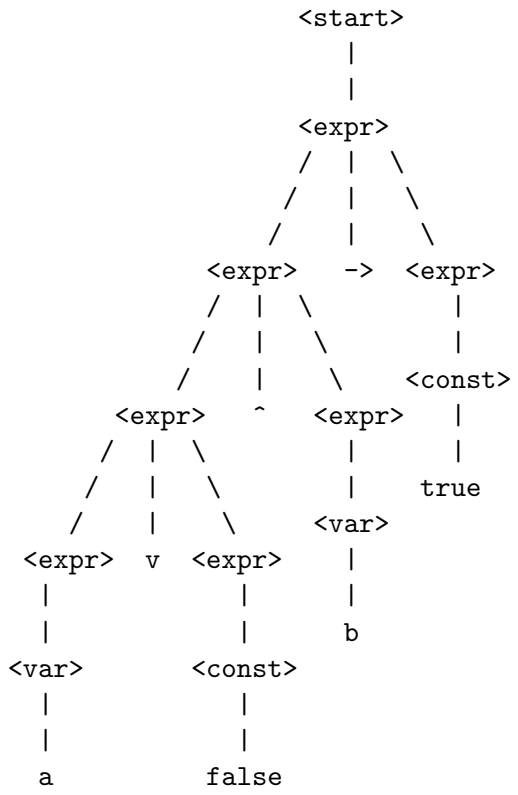
   $\mathsf{a} \vee \mathsf{false} \wedge \mathsf{b} \rightarrow \mathsf{true}$.

<start>
$\Rightarrow_{LM}$ <expr>
$\Rightarrow_{LM}$ <expr> $\vee$ <expr>
$\Rightarrow_{LM}$ <var> $\vee$ <expr>
$\Rightarrow_{LM}$ a $\vee$ <expr>
$\Rightarrow_{LM}$ a $\vee$ <expr> $\wedge$ <expr>
$\Rightarrow_{LM}$ a $\vee$ <const> $\wedge$ <expr>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ <expr>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ <expr> $\rightarrow$ <expr>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ <var> $\rightarrow$ <expr>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ b $\rightarrow$ <expr>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ b $\rightarrow$ <const>
$\Rightarrow_{LM}$ a $\vee$ false $\wedge$ b $\rightarrow$ true

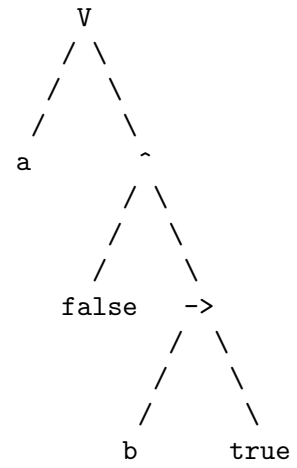
$$
\begin{aligned}
&\langle \text{start} \rangle \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \rightarrow \langle \text{expr} \rangle \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \rightarrow \langle \text{const} \rangle \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \wedge \langle \text{expr} \rangle \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \wedge \langle \text{var} \rangle \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \wedge \text{b} \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \vee \langle \text{expr} \rangle \wedge \text{b} \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \vee \langle \text{const} \rangle \wedge \text{b} \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{expr} \rangle \vee \text{false} \wedge \text{b} \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\langle \text{var} \rangle \vee \text{false} \wedge \text{b} \rightarrow \text{true} \\
\Rightarrow_{RM}\ &\text{a} \vee \text{false} \wedge \text{b} \rightarrow \text{true}
\end{aligned}
$$

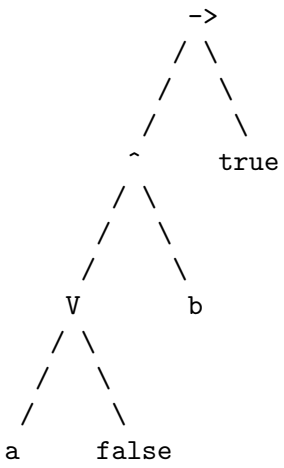
2. Show the corresponding parse trees for the derivations





3. Show the corresponding AST.





4. Show that the above grammar is ambiguous.

   Since the sentence "a $\vee$ false $\vee$ b $\rightarrow$ true" has multiple possible parse trees, the grammar is ambiguous. Many other sentences also demonstrate this.

5. Give an unambiguous grammar for the same language that enforces the following precedence and associativity:

   - $\wedge$ has highest precedence (binds strongest), followed by $\vee$, and then $\rightarrow$
   - $\wedge$ and $\vee$ are left associative, and $\rightarrow$ is right associative

   $< start >::=< expr >$

   $< expr >::=< orexpr > \rightarrow < expr > \mid < orexpr >$

   $< orexpr >::=< orexpr > \vee < andexpr > \mid < andexpr >$

   $< andexpr >::=< andexpr > \wedge < term > \mid < term >$

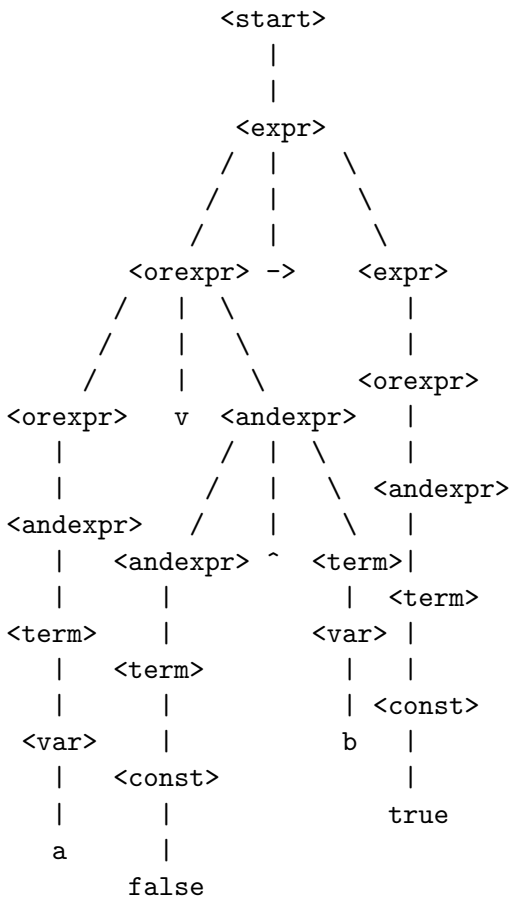
   $< term >::=< const > \mid < var >$

$< const >::= true \mid false$

$< var >::= a \mid b \mid c \mid \ldots \mid z$

6. Give the parse tree and AST for your new, unambiguous grammar for the sentence in 1.

   The parse tree is shown below:



The AST is shown below: