

**Math 1330 Winter Mini Semester  
Midterm**

**There will be approximately 25-28 questions; some of them will be multiple choice, some will be free response. You may get partial credit on the free response questions if your work is correct.**

**Time: 2hours+ (depending on the number of questions)**

**Where and When: Check calendar**

**What to bring: Picture ID, writing utensils.**

**No calculators.**

**A formula sheet will be included; you can not bring anything.**

The following formulas will be provided. It is your responsibility to locate the formula sheet before you start your test. If you can't find it, ask proctors for help.

### Handy Formulas

$$\sin(s + t) = \sin s \cos t + \cos s \sin t$$

$$\sin(s - t) = \sin s \cos t - \cos s \sin t$$

$$\cos(s + t) = \cos s \cos t - \sin s \sin t$$

$$\cos(s - t) = \cos s \cos t + \sin s \sin t$$

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

$$\sin(2t) = 2 \sin t \cos t$$

$$\cos(2t) = \cos^2 t - \sin^2 t$$

$$\sin \frac{s}{2} = \pm \sqrt{\frac{1 - \cos s}{2}}$$

$$\cos \frac{s}{2} = \pm \sqrt{\frac{1 + \cos s}{2}}$$

$$\tan \frac{s}{2} = \frac{\sin s}{1 + \cos s}$$

**Example 1a:** Let  $f(x) = \frac{x^2 - x - 12}{x^2 - 2x - 3}$  be a given rational function.

a) Domain:

b) Vertical Asymptote(s):

c) Hole:

d) Horizontal Asymptote:

e) Does the graph intersect the HA? If so, what is the x-coordinate of the intersection?

**Example 1b:**  $f(x) = \frac{x - 4}{x^2 + 3x - 4}$

a) Domain:

b) Vertical Asymptote(s):

c) Hole:

d) Horizontal Asymptote:

e) Does the graph intersect the HA? If so, what is the x-coordinate of the intersection?

**Example 2:** For the following functions, find the difference quotient  $\frac{f(x+h) - f(x)}{h}$  and simplify the difference quotient for  $x = 1$ .

a)  $f(x) = 2x^2 - 7x + 4$

b)  $f(x) = \frac{3}{x+2}$

**Example 3:** Given  $f(x) = 4\ln(x)$  and  $g(x) = e^{3x}$ , find the following:

a)  $(f \circ g)(3) =$

b)  $(g \circ f)(2) =$

**Example 4:** A motorcycle has wheels with a 12 inch radius. If each wheel's rate of turn is 4 revolutions per second, how fast is the car moving in units of inches/sec?

**Example 5:** Find the area of the sector if

a)  $r = 4, \theta = \frac{\pi}{6}$

b)  $r = 8, \theta = 60^\circ$

**Example 6:** Simplify the following expressions:

a)  $\frac{\sin(\theta)}{1 - \cos(\theta)} - \frac{\sin(\theta)}{1 + \cos(\theta)}$

b)  $(1 - \sin \theta)(\sec \theta + \tan \theta)$

**Example 7:** Evaluate

a)  $\tan\left(\frac{5\pi}{4}\right) + \sin\left(\frac{11\pi}{6}\right)$

b)  $\cot\left(\frac{5\pi}{6}\right) + \cos\left(\frac{2\pi}{3}\right)$

**Example 8:** Suppose that  $\theta$  is an acute angle of a right triangle and that  $\tan(\theta) = \frac{3\sqrt{2}}{5}$ .

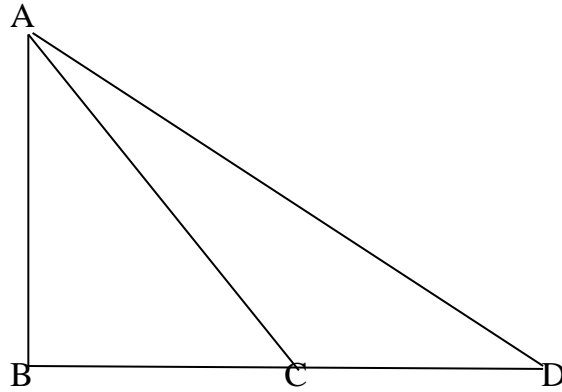
a) Find  $\sin(\theta)$  and  $\cos(\theta)$ .

b) Evaluate  $1 - \cos^2(\theta)$ .

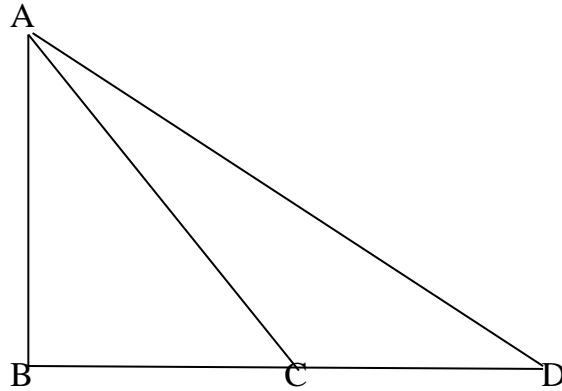
c) Evaluate  $1 - \sin^2(\theta)$ .

**Example 9:**

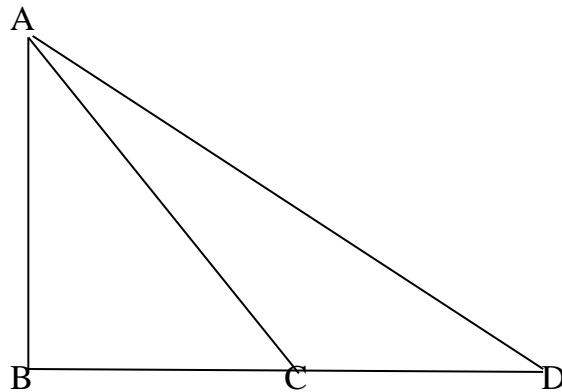
- a) In the figure below, angle B is a right angle,  $m(D)=45$  and  $m(ACB)=60$ . If  $AC=10$ , find the length of  $AD$ .



- b) In the figure below, angle B is a right angle,  $m(D)=30$  and  $m(ACB)=60$ . If  $AC=5$ , find the length of  $AD$ .



- c) In the figure below, angle B is a right angle,  $m(D)=30$  and  $m(ACB)=45$ . If  $AC=8$ , find the length of  $AD$ .





**Example 10:** Given a triangle ABC, with B being a right angle, and AB =7 and AC=12, find all six trigonometric functions for angle C.

**Example 11:** Evaluate the following if possible:

a)  $\tan\left(\frac{5\pi}{3}\right)$

b)  $\cot\left(\frac{5\pi}{6}\right)$

c)  $\cos\left(\frac{2\pi}{3}\right)$

d)  $\sin\left(-\frac{5\pi}{3}\right)$

e)  $\csc\left(\frac{4\pi}{3}\right)$

f)  $\sec\left(\frac{7\pi}{6}\right)$

g)  $\sin\left(\frac{11\pi}{6}\right)$

h)  $\cot\left(\frac{3\pi}{4}\right)$

i)  $\tan\left(\frac{5\pi}{4}\right)$

j)  $\cos\left(-\frac{7\pi}{4}\right)$

k)  $\tan(135^\circ)$

l)  $\cos(360^\circ)$

m)  $\tan(-45^\circ)$

n)  $\cot(-225^\circ)$

o)  $\tan(-315^\circ)$

p)  $\sin(-45^\circ)$

q)  $\cos(-60^\circ)$

r)  $\cos(300^\circ)$

s)  $\sin(240^\circ)$

t)  $\sin(210^\circ)$

**Example 12:** Let  $P(x,y)$  denote the point where the terminal side of an angle  $\theta$  meets the unit circle.

a) If  $P$  is in Quadrant II and  $y=3/4$ , find

$\sec(\theta)$

$\cot(\theta)$

b) If  $P$  is in Quadrant IV and  $x=2/5$ , find

$\csc(\theta)$

$\cot(\theta)$

1) Given  $\tan(x) = -\frac{1}{5}$ ,  $90^\circ < x < 180^\circ$ ,

a)  $\sin(2x)$

b)  $\cos(2x)$

2) Evaluate

a)  $\sin\left(\frac{11\pi}{3}\right) \cos\left(\frac{-31\pi}{6}\right) \tan\left(\frac{41\pi}{4}\right)$

b)  $\cos(40\pi) + \sin(21\pi) + \tan\left(\frac{43\pi}{4}\right) = ?$

3) Solve the following equation on the interval  $[0, 2\pi)$ :

$$4 \sin(x) + 7 = 5$$

4) Find the exact value of the following expression:

a)  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) + \tan^{-1}(\sqrt{3})$

b)  $\arccos\left(-\frac{\sqrt{2}}{2}\right) + \arcsin\left(-\frac{\sqrt{2}}{2}\right) + \arctan(-1)$

c)  $\arctan(0) + \arctan(1) = ?$

5) Find the exact value of the following expression:

a)  $\sec\left(\tan^{-1}\left(\frac{2}{5}\right)\right)$

b)  $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

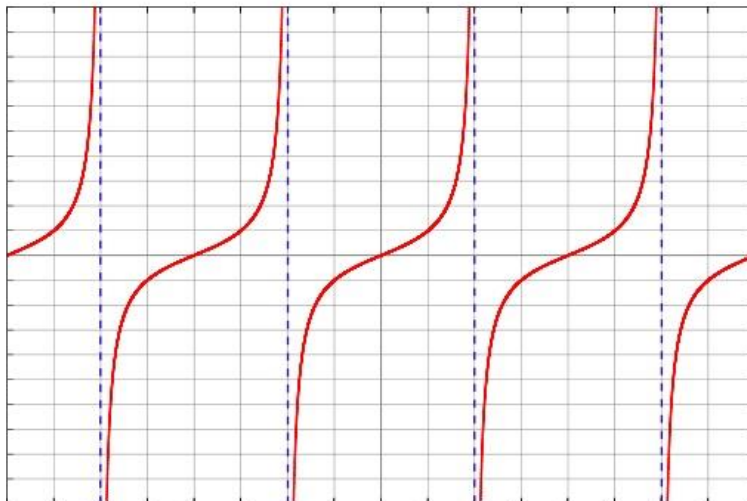
6) Find the vertical asymptotes for the following functions:

a)  $f(x) = 2 \csc\left(x + \frac{\pi}{4}\right)$

7) a) Write a sine function with amplitude 10, horizontal shift 5 to the left, vertical shift 2 down, and period 5.

b) Write a cosine function with amplitude 6, horizontal shift 3 to the right, vertical shift 4 up, and period  $\frac{\pi}{3}$ .

8) Given the following graph, find the formula for it:  
The point C(1/8, 2) is on this graph;  $x = 1/4$  and  $x = -1/4$  are asymptotes.



9) Write the equivalent form

a) 
$$\frac{1-2\sin^2(x)}{6\sin(-x)\cos(-x)}$$

b) 
$$\frac{-2\cos^2(x)+1}{8\sin(-x)\cos(-x)}$$

c) 
$$4\sin(-t+20\pi)\cos(-t+12\pi)$$



10) Given  $\sin(x) = \frac{1}{4}$ ,  $90^\circ < x < 180^\circ$ , and  $\sin(y) = -\frac{2}{5}$ ,  $180^\circ < y < 270^\circ$ , find:

a)  $\sin(x - y)$

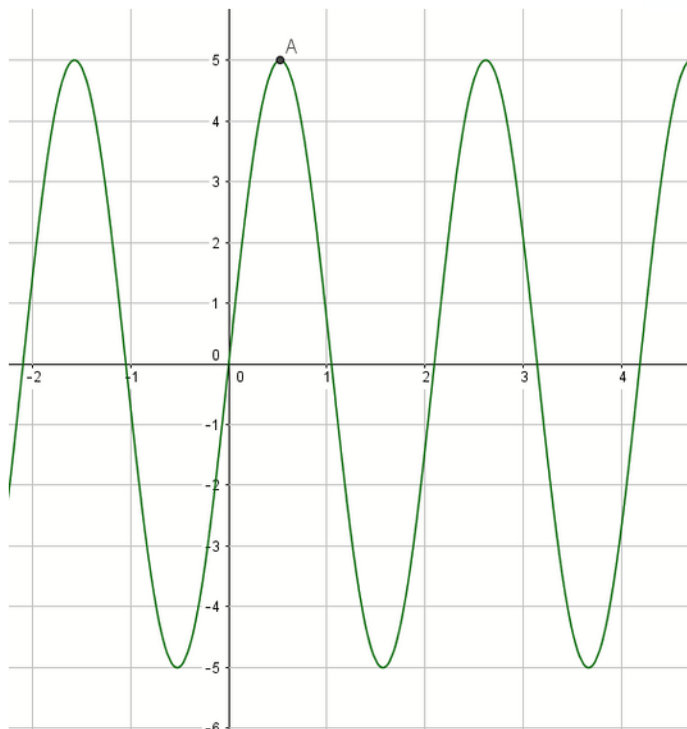
b)  $\sin(x + y)$

c)  $\cos(x - y)$

d)  $\cos(x + y)$

11) Find an equation for the sine function passing through (0,0) given that the first maximum point on the right of the origin is

$$A \left( \frac{\pi}{6}, 5 \right)$$



12) a) Find all solutions of the equation  $4 \cos(x) + 5 = 3$  over the interval  $[0, 2\pi)$ .

b) Find all solutions of the equation  $2 \sin(x) + 3\sqrt{2} = 4\sqrt{2}$  over the interval  $[0, 2\pi)$ .

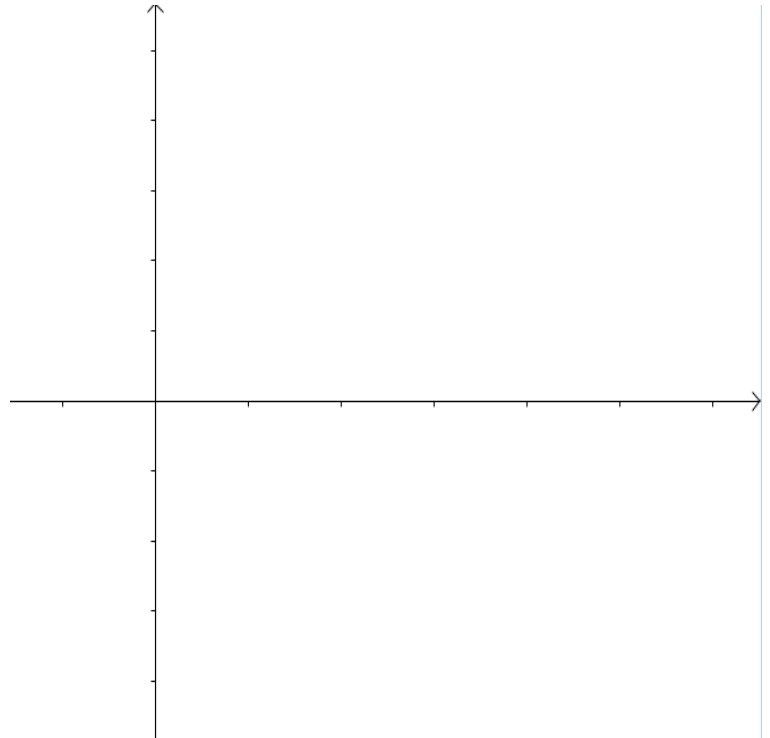
13) a)  $f(x) = 5 \sin(6x)$

What is the domain of this function?

What is the range of this function?

Graph the function over one period.

Label the intercepts, minimum/maximum value(s) with an ordered pair.



b)  $f(x) = 6 \cos(2x)$

What is the domain of this function?

What is the range of this function?

Graph the function over one period.

Label the intercepts, minimum/maximum value(s) with an ordered pair.

