Engineering Mathematics One (ENGE501): Differential and Integral Calculus (MATH501)

Assignment Four

Due Monday 29 May 2017

Please ensure that all your working is clearly set out, all pages are stapled together, and that **your name** along with the **course name**, your **lecturer name** and your **stream number** (ex. ENGE501/10 etc. or MATH501/10) appears on the front page of the assignment. Please place completed assignments in the assignment box in WT level 1 by **4:30pm** on the due date.

Name	
Student ID	
Course name	
Paper Code/Stream	
Lecturer's Name	

Questions:

1. Evaluate each of the following definite integrals:

$$\int_1^4 \frac{1}{\sqrt[3]{x}}\,dx \qquad \int_0^1 \frac{1}{7-2t}\,dt$$
$$\int_0^5 \left(2e^{3x} - 4\cos 2x\right)\,dx \qquad \int_0^1 \frac{1}{(7-2t)^2}\,dt$$

2. Consider the square with vertices (0,0), (0,1), (1,1) and (1,0). The parabola $y = x^2$ splits the square into two unequal parts. Find the ratio of the smaller area to the larger one.

3. Evaluate each of the following indefinite integrals:

$$\int x\left(1+3x^5\right)\,dx \qquad \int x^4\left(1+3x^5\right)\,dx$$
$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad \int \frac{5x^2-4x+7}{(x^2+1)(2x-1)}\,dx$$

4. Evaluate the improper integral $\int_0^1 \frac{5x^2-4x+7}{(x^2+1)(2x-1)}\,dx$.

5. The force acting on a particle on the x-axis is

$$F(x) = \frac{9800}{(x+500)^2}, \qquad when \qquad x \geq 0$$

   (a) Find the work required to move the particle from $x = 0$ to $x = 20$.

   (b) By evaluating an improper integral, find the amount of work required to move the particle from $x = 0$ to $x = \infty$

6. A heavy chain that weighs 8 kilograms/metre is initially hanging 20 metres down into a mine shaft. How much work is done in pulling up the chain to the top of the mine shaft? (You can take $g = 9.81\text{ms}^{-2}$).

7. The rate at which a pollutant is being dumped into a small stream at time t is given by $P_0 e^{-kt}$, where P_0 is the rate of the pollutant initially released into the stream. Suppose $P_0 = 1000$ and $k = 0.06$. Find the total amount of the pollutant that will be released into the stream into the indefinite future.

8. When mixing a certain cocktail a barperson starts with a 5 litre solution of 10% alcohol. Into this solution the barperson mixes vodka (40% alcohol) at the rate of 2 litres/minute and pours into glasses at the rate of 3 litres/minute.

   (a) Find a formula for the volume $V(t)$ of the mixture at time t minutes.

   (b) Find a differential equation that tells the rate of change of the amount of alcohol $A(t)$ at time t minutes.

   (c) Solve the differential equation and use the initial condition to find a formula for $A(t)$

9. When a charged capacitor is connected to earth, it discharges and the voltage V across it will decrease according to the equation:

$$\frac{dV}{dt} = \frac{-V}{RC}$$

   where R (the resistance) and C (the capacitance) are constants. If the voltage V across a certain capacitor with capacitance $C = 3 \times 10^{-6}$ farads drops from 10 volts to 1 volt in two seconds, determine the resistance R.