

Homework 3

Textbook Chapter 3 problems 30, 31, 38, 48, 49, 72, 82,

$m = \frac{500}{386.4} \text{ lb-sec}^2/\text{in}$, $F(t) = 200 \sin 100 \pi t \text{ lb}$. Let $X_{\max} = 0.05 \text{ in} < 0.1 \text{ in}$ (maximum permissible value). From Eq. (3.33),

$$X_{\max} = \delta_{\text{st}} \frac{1}{2 \zeta \sqrt{1 - \zeta^2}} = 0.05 \quad (1)$$

Let $\zeta = 0.01$. Then $\delta_{\text{st}} = \frac{F_0}{k_{\text{eq}}} = \frac{200}{k_{\text{eq}}}$ and Eq. (1) gives

$$k_{\text{eq}} = \frac{200}{2 (0.01) \sqrt{1 - 0.0001} (0.05)} = 20.0020 (10^4) \text{ lb/in}$$

Since shock mounts are in parallel, stiffness of each mount $= k = \frac{k_{\text{eq}}}{3} = 6.6673 (10^4) \text{ lb/in}$.

$$\zeta = \frac{c_{\text{eq}}}{c_c} = \frac{c_{\text{eq}}}{\sqrt{2 k_{\text{eq}} m}}$$

$$\text{or } c_{\text{eq}} = \zeta \sqrt{2 k_{\text{eq}} m} = 0.01 \sqrt{2 (20.0020 (10^4)) \left(\frac{500}{386.4} \right)} = 7.1948 \text{ lb-sec/in}$$

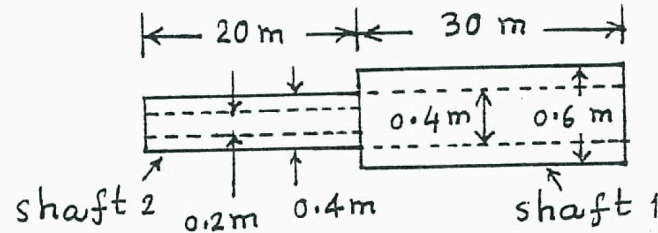
and hence $c = \frac{c_{\text{eq}}}{3} = 2.3983 \text{ lb-sec/in}$

3.31

Equation of motion for torsional system:

$$J_0 \ddot{\theta} + c_t (\dot{\theta} - \dot{\alpha}) + k_t (\theta - \alpha) = 0 \quad (1)$$

where θ = angular displacement of shaft and α = angular displacement of base of shaft
 $= \alpha_0 \sin \omega t$. Steady state response of propeller (Eq. (3.67)):



$$\theta_p(t) = \Theta \sin(\omega t - \phi) \quad (2)$$

$$\text{where } \Theta = \alpha_0 \left\{ \frac{k_t^2 + (c_t \omega)^2}{(k_t - J_0 \omega^2)^2 - (c_t \omega)^2} \right\}^{\frac{1}{2}} \quad (3)$$

$$\text{and } \phi = \tan^{-1} \left\{ \frac{J_0 c_t \omega^3}{k_t (k_t - J_0 \omega^2) + (c_t \omega)^2} \right\} \quad (4)$$

Here $J_0 = 10^4 \text{ kg-m}^2$, $\zeta_t = 0.1$, and $\omega = 314.16 \text{ rad/sec}$. Torsional stiffnesses of shafts:

$$(k_t)_1 = \frac{G_1 J_1}{\ell_1} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.6^4 - 0.4^4) \right)}{30} = 27.2272 (10^6) \text{ N-m/rad}$$

$$(k_t)_2 = \frac{G_2 J_2}{\ell_2} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.4^4 - 0.2^4) \right)}{20} = 9.4248 (10^6) \text{ N-m/rad}$$

← Function like these will be given in the exam

Series springs give:

$$k_t = \frac{(k_t)_1 (k_t)_2}{(k_t)_1 + (k_t)_2} = \frac{(27.2272 (10^6)) (9.4248 (10^6))}{27.2272 (10^6) + 9.4248 (10^6)} = 7.0013 (10^6) \text{ N-m/rad}$$

$$c_t = \zeta (2 \sqrt{J_0 k_t}) = 0.1 (2) \sqrt{(10^4) (7.0013 (10^6))} = 52,919.8624 \text{ N-m-s/rad}$$

From Eq. (3),

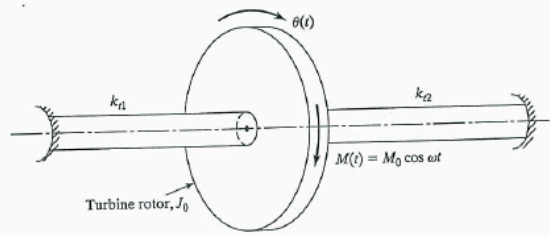
$$\Theta = 0.05 \left[\frac{(7.0013 (10^6))^2 + \left\{ 5.2920 (10^4) (314.16^2) \right\}^2}{\left\{ 7.0013 (10^6) - (10^4) (314.16^2) \right\}^2 + \left\{ 5.2920 (10^4) (314.16) \right\}^2} \right]^{\frac{1}{2}}$$

$$= 9.2028 (10^{-4}) \text{ rad}$$

$$\phi = \tan^{-1} \left\{ \frac{(10^4) (5.2920 (10^4)) (314.16^3)}{7.0013 (10^6) \left[7.0013 (10^6) - (10^4) (314.16^2) \right] + (5.2920 (10^4) (314.16))^2} \right\}$$

$$= \tan^{-1} (59.3664) = 89.0350^\circ = 1.5540 \text{ rad}$$

3.38



$$M_t = M_0 \cos \omega t, \quad k_t = k_{t1} + k_{t2}$$

Equation of motion:

$$J_0 \ddot{\theta} + c_t \dot{\theta} + (k_{t1} + k_{t2}) \theta = M(t) = M_0 \cos \omega t \quad (1)$$

For the given data, Eq. (1) becomes

$$0.05 \ddot{\theta} + 2.5 \dot{\theta} + 7000 \theta = 200 \cos 500t \quad (2)$$

Steady state response of the turbine rotor can be expressed, similar to Eqs. (3.25), (3.28) and (3.29) for a torsional system, as

$$\theta_p(t) = \Theta \cos(\omega t - \phi) \quad (3)$$

where

$$\Theta = \frac{M_0}{\{(k_t - J_0 \omega^2)^2 + c_t^2 \omega^2\}^{\frac{1}{2}}} \quad (4)$$

and

$$\phi = \tan^{-1} \left(\frac{c_t \omega}{k_t - J_0 \omega^2} \right) \quad (5)$$

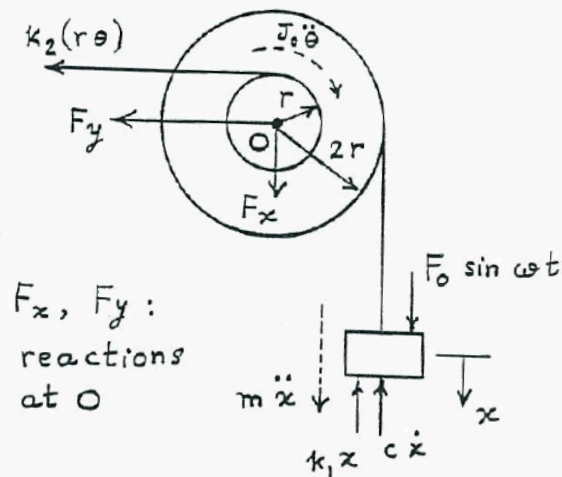
For the given data,

$$J_0 = 0.05, \quad M_0 = 200, \quad k_t = 7000, \quad c_t = 2.5, \quad \omega = 500$$

Hence Eqs. (4) and (5) give

$$\begin{aligned} \Theta &= \frac{200}{[(7000 - 0.05 \times 25 \times 10^4)^2 + (2.5)^2 (25 \times 10^4)]^{\frac{1}{2}}} \\ &= 6.2868 \times 10^{-6} \text{ rad} \end{aligned}$$

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{2.5 \times 500}{7000 - 0.05 \times 250000} \right) \\ &= \tan^{-1} \left(-\frac{1250}{5500} \right) = \tan^{-1} (-0.2273) \\ &= -12.8043^\circ = -0.2235 \text{ rad} \end{aligned}$$



Equation of motion for rotation of pulley about O:

$$-k_2(\theta r)r - J_0 \ddot{\theta} - k_1 x(2r) - c \dot{x}(2r) + F_0 \sin \omega t(2r) - m \ddot{x}(2r) = 0 \quad (1)$$

where $\theta = x/(2r)$. Equation (1) can be rearranged as:

$$\left(\frac{J_0}{2r} + 2mr \right) \ddot{x} + 2cr \dot{x} + \left(2k_1 r + \frac{1}{2} k_2 r \right) x = 2r F_0 \sin \omega t \quad (2)$$

For given data, Eq. (2) becomes

$$11 \ddot{x} + 50 \dot{x} + 112.5 x = 5 \sin 20 t \quad (3)$$

Steady state response is given by Eq. (3.25):

$$x_p(t) = X \cos(\omega t - \phi)$$

$$\text{where } X = \frac{5}{\left[\left\{ 112.5 - 11(20^2) \right\}^2 + \left\{ 50(20) \right\}^2 \right]^{\frac{1}{2}}} = 0.001136 \text{ m}$$

$$\text{and } \phi = \tan^{-1} \left(\frac{50(20)}{112.5 - 11(20^2)} \right) = -0.2291 \text{ rad} = -13.1287^\circ$$

(a)

$$\begin{aligned} \sum M_0 &= 0 \text{ (about hinge):} \\ I_0 \ddot{\theta} + \left(k \ell \frac{3\ell}{4} \right) \frac{3\ell}{4} + (c \ell \dot{\theta}) \ell &= \frac{\ell}{2} F_0 \sin \omega t \\ \text{or } I_0 \ddot{\theta} + c \ell^2 \dot{\theta} + \frac{9}{16} k \ell^2 \theta &= \frac{F_0 \ell}{2} \sin \omega t \end{aligned}$$

Magnitude of steady state response:

$$\Theta_a = \left(\frac{F_0 \ell}{2} \right) / \left[\left\{ \frac{9}{16} k \ell^2 - I_0 \omega^2 \right\}^2 + (c \ell^2 \omega)^2 \right]^{\frac{1}{2}} \quad (1)$$

(b)

$$\begin{aligned} \sum M_0 &= 0 \text{ (about hinge):} \\ I_0 \ddot{\theta} + (k \ell \theta) \ell + \left(c \frac{3\ell}{4} \dot{\theta} \right) \frac{3\ell}{4} &= \frac{\ell}{2} F_0 \sin \omega t \\ \text{or } I_0 \ddot{\theta} + \frac{9}{16} c \ell^2 \dot{\theta} + k \ell^2 \theta &= \frac{F_0 \ell}{2} \sin \omega t \end{aligned}$$

Magnitude of steady state response:

$$\Theta_b = \left(\frac{F_0 \ell}{2} \right) / \left[\left\{ k \ell^2 - I_0 \omega^2 \right\}^2 + \left\{ \frac{9}{16} c \ell^2 \omega \right\}^2 \right]^{\frac{1}{2}} \quad (2)$$

Usually, c is small compared to k . If the term containing c is negligible, Θ_a will be smaller than Θ_b . Hence arrangement (a) is desirable.

k = spring constant of cantilever beam

$$= \frac{3EI}{l^3} = \frac{3(2.5 \times 10^6)}{4^3}$$

$$= 0.1172 \times 10^6 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m_1 + 0.25 m_b}} = \sqrt{\frac{0.1172 \times 10^6}{20 + 0.25(240)}} = 38.2753 \text{ rad/sec}$$

$$\omega = 2\pi(1500)/60 = 157.08 \text{ rad/sec}$$

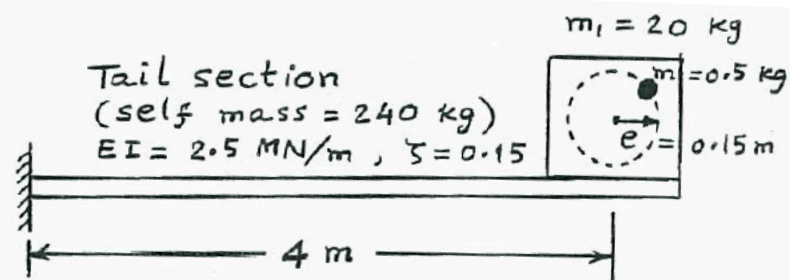
$$r = \omega/\omega_n = 157.08/38.2753 = 4.1040, \quad r^2 = 16.8428$$

Forced response is given by Eq. (3.79):

$$x_p(t) = X \sin(\omega t - \phi)$$

where

$$\begin{aligned} X &= \frac{m e}{m_1} \cdot \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \\ &= \frac{(0.5)(0.15)}{20} \cdot \frac{16.8428}{\sqrt{(1-16.8428)^2 + (2 \times 0.15 \times 4.1040)^2}} \\ &= 3.9747 \times 10^{-3} \text{ m} = 3.9747 \text{ mm} \end{aligned}$$



$$\omega_{n, fan} = \sqrt{\frac{k}{m_{fan}}}$$

$$= \sqrt{\frac{200}{50/386.4}}$$

$$= 39.3141 \text{ rad/sec}$$

$$\omega = \frac{2\pi(750)}{60} = 78.54 \text{ rad/sec}$$

$$(J_p)_{plate + fan} = \frac{1}{3} \left(\frac{100}{386.4} \right) (40)^2 + \left(\frac{50}{386.4} \right) (5)^2 = 141.2612 \text{ lb-in-sec}^2$$

$$F_0 = m e \omega^2 = \left(\frac{50}{386.4} \right) (0.1) (78.54)^2 = 79.8205 \text{ lb}$$

Point R is subjected to the force, $F(t) = F_0 \cos \omega t = 79.8205 \cos 78.54 t$
Assume that S is not moving.

Then R is displaced by:

$$x(t) = \frac{F_0 \cos \omega t}{|k - m \omega^2|} = \frac{F_0 \cos \omega t}{k \left| 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right|} = \frac{79.8205 \cos \omega t}{200 \left| 1 - \left(\frac{78.54}{39.3141} \right)^2 \right|}$$

$$= 0.1334 \cos 78.54 t \text{ inch}$$

Let θ = angular displacement of plate PQ.

Displacement of S = 5θ inch

Extension of spring RS = $(5\theta - 0.1334 \cos 78.54 t)$ inch

Restoring moment of spring force about P

$$= 200 [5\theta - 0.1334 \cos 78.54 t] 5 \text{ lb-in}$$

Velocity of Q = $40 \dot{\theta}$ inch/sec

Damping force at Q = $40 \dot{\theta} (1) = 40 \dot{\theta}$ lb

Moment of damping force about P = $40 \dot{\theta} (40) = 1600 \dot{\theta}$ lb-in

Equation of motion of plate PQ:

$$J_p \ddot{\theta} + 1600 \dot{\theta} + 1000 (5\theta - 0.1334 \cos 78.54 t) = 0$$

$$\text{i.e., } 141.2612 \ddot{\theta} + 1600 \dot{\theta} + 5000 \theta = 133.4 \cos 78.54 t \quad (E_1)$$

Comparing (E_1) with $E_g(3.24)$, the solution of (E_1) can be expressed as $\theta_p(t) = \Theta \cos(\omega t - \phi)$

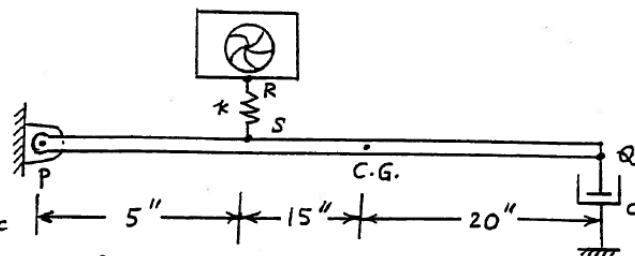
where, from $E_g(3.30)$ and (3.31) , we get

$$\Theta = \frac{(133.4/5000)}{\sqrt{(1 - 174.2751)^2 + (2 \times 0.9519 \times 13.2013)^2}} = 1.5238 \times 10^{-4} \text{ rad}$$

$$\text{and } \phi = \tan^{-1}(-25.1326/173.2751) = -8.2529^\circ$$

Steady state motion of Q = $\theta_p(40)$

$$= 0.006095 \cos(78.54 t + 8.2529^\circ) \text{ inch}$$



Displacement of $S = \oplus(5)$ inch

$$= (1.5238 \times 10^{-4})(5) \text{ inch}$$

$$= 0.0007619 \text{ inch}$$

$D_2 D_3$ = maximum deformation of
Spring ≈ 0.1334 "

Max. force transmitted to point $S = k(D_2 D_3)$

$$= 200(0.1334) = 26.68 \text{ lb}$$

