Student understanding of the work-energy and impulse-momentum theorems

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Student understanding of the impulse-momentum and work-energy theorems was assessed by performance on tasks requiring the application of these relationships to the analysis of an actual motion. The participants in the study were undergraduates enrolled in either the honors section of a calculus-based introductory physics course or in the regular algebra-based course. The students were asked to compare the changes in momentum and kinetic energy of two frictionless dry-ice pucks as they moved rectilinearly under the influence of the same constant force. The results of the investigation revealed that most of the students were unable to relate the algebraic formalism learned in class to the simple motion that they observed.

I. INTRODUCTION

This paper reports the results of an investigation of student understanding of the concepts of impulse and work and the relationship of these concepts to changes in momentum and kinetic energy.[1] Research over the past several years has provided a substantial amount of detail on the difficulty students have in making the proper connection between force and motion. Considerably less information is available about their ability to relate force to more complex concepts.[2] The present study is part of the ongoing effort by the Physics Education Group at the University of Washington to identify specific difficulties encountered by students in various topics in physics and to use these findings as a guide in designing instruction.[3]

The aspect of understanding emphasized in the investigation is the ability to apply the impulse-momentum and work-energy theorems to the analysis of an actual motion. We wanted to determine if students who had studied the relevant concepts could make a correspondence between an observed motion and the algebraic formalism. As in much of our research, the method used is the individual demonstration interview. Because of its focus on real objects and events, we have found this technique to be particularly effective for examining the ability of students to make connections between the physical world and its algebraic and graphical representations.

A typical interview begins with a simple demonstration that serves as the basis for a set of tasks to be performed by the student. The tasks are accompanied by questions that have been structured to reveal the meaning the student ascribes to a particular concept or relation. The questions become part of a dialogue in which the investigator attempts to probe the student's thinking. In addition to those that are prescribed, the investigator may ask additional questions to clarify a reply or follow up on a comment. The student's actions and other nonverbal responses are noted. The entire discussion is audiotaped and transcribed. The transcripts, together with the investigator's notes from the interviews, are later analyzed in detail.

The 28 students who participated in the investigation were volunteers from two introductory physics courses at the University of Washington. Sixteen students were enrolled in the noncalculus physics course and 12 were in the honors section of calculus-based physics. For each group, the average of the final course grades of the participants in the study was somewhat higher than the average for the respective groups as a whole.

II. DESCRIPTION OF THE TASKS

In the tasks used in this investigation, students are asked to compare the changes in momentum and kinetic energy of two dry-ice pucks that move on a glass table, as shown in Fig. 1. The table is about 2 m long and 1 m wide. Two parallel lines, labeled (a) and (b), mark off a distance of about 30 cm on the table. The two pucks differ greatly in mass but are subjected to the same constant force. One puck is made of brass and has a mass of about 3500 g; the other, made of plastic and aluminum, has a mass of only about 350 g. The diameter of the base is about 15 cm for the brass puck and about 10 cm for the plastic one. The height of both pucks is about 15 cm and the diameter of both dry-ice container sections is about 7 cm.

During the demonstration, each puck is started from rest just behind line (a), moves rectilinearly under a constant force applied between lines (a) and (b), and then moves freely beyond line (b). Although the motion of each puck is observed separately, it is readily apparent that the brass puck traverses the distance between lines (a) and (b) much more slowly than the plastic one.

The force on the pucks is supplied by a steady stream of air blown through the hose of a reversed vacuum cleaner of the type used in air track experiments. The magnitude of the applied force can be varied by moving the end of the

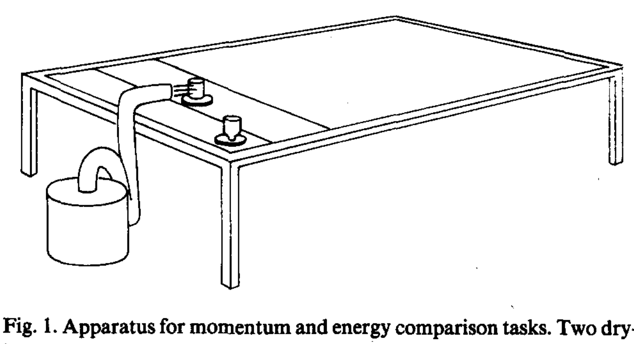

Fig. 1. Apparatus for momentum and energy comparison tasks. Two dry-ice pucks move without friction on a level glass table. Equal force is applied to the two pucks by a reversed vacuum cleaner as they move rectilinearly from line (a) to line (b).

hose toward or away from the puck. Attached to the opening are small strips of paper that, when blown out by the air, serve as spacers for maintaining a constant separation between the hose and the puck. By placing a hand in front of the opening, the students can feel that the force is constant as long as the distance from the hose remains unchanged.

Each of the two tasks presented to the students consists essentially of a single question. After observing the motion of both pucks, the student is asked whether the pucks have the same or different momentum during their free-motion after crossing line (b). Regardless of the response to this first question, the student is then asked if the two pucks have the same or different kinetic energy during their free-motion beyond line (b). If the student does not reply correctly to either or both questions, the investigator begins a dialogue that provides an increasing amount of help to the student in analyzing the motion.

To make a correct comparison of the changes in momentum (Δp) and kinetic energy (ΔT) for the two pucks, it is necessary to know how these quantities are related to the concepts of impulse and work. The honors students were familiar with the impulse-momentum and work-energy theorems in integral form. For motion in one dimension,

$$\int_i^f F_x\, dt = \Delta p_x = mv_f - mv_i$$

and

$$\int_i^f \mathbf{F}\cdot d\mathbf{x} = \Delta T = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2,$$

where $\mathbf{F}$ is the total force acting on the puck, m is the mass, and v_i and v_f are the initial and final velocities.

The noncalculus physics students had encountered these relationships in terms of $F\,\Delta t$ and $F\,\Delta x$, the forms to which they can be reduced when the force is constant: the situation in the demonstration.

Successful performance on the tasks requires only qualitative reasoning. Before the students are asked to compare the momenta and kinetic energies of the two pucks, they are led to assume (as is approximately the case) that the air stream exerts the same force on the two pucks. Since equal constant forces are applied to both pucks, the change in momentum is proportional to the time each takes to traverse the distance between the lines. Because of its greater mass, a smaller acceleration is imparted to the brass puck. During the longer time it spends between the lines, it receives a greater impulse. Hence the brass puck experiences a greater change in momentum than the plastic puck.

Comparing the kinetic energies requires fewer steps than comparing the momenta. Since the pucks move almost without friction and do not rotate, the total change in kinetic energy of the center of mass is equal to the work done on the puck. Since the same constant force is applied to each puck for the same distance, the change in kinetic energy is the same.

In order for a response to be considered correct, it was necessary for the student both to make the right comparison and to give the proper reasoning. Students who concluded that the brass puck had the larger momentum or that the kinetic energies were the same, but who did not give adequate justification, are not included in the category of students who made correct comparisons. If a student could not decide if the momenta or kinetic energies were the same or different, or if the student's justification of his comparison could not be deciphered or adequately clarified, the response was considered indeterminate. Students who gave indeterminate responses generally appeared more perplexed by the tasks than students who gave incorrect or inadequately justified responses.

Although the students who participated in the investigation had all completed the parts of introductory mechanics on momentum and energy, it was not really expected that many would be able to make a correct analysis on observing the demonstration for the first time. Therefore, as the interview progressed, the students were given an increasing amount of guidance toward noting the important features of the motion that were needed to make the comparison. If the initial response of a student on either task was incorrect or inadequately justified, the investigator would draw the student's attention to the way in which the pucks had been set in motion, i.e., the same constant force applied for the same distance. It was expected that once the students had taken note of these deliberately orchestrated conditions they would make use of the concepts of impulse and work in analyzing the motion. If, with this amount of prompting, a student still seemed at a loss about how to approach the task, the student was asked directly if he was familiar with the terms "impulse" and "work" and if the words represented ideas that could be applied to the demonstration at hand. If at this point the student was still unable to make a proper comparison of the momenta or kinetic energies of the pucks, the interview was terminated.

III. PERFORMANCE ON THE TASKS

The results of the momentum comparison task are summarized in Tables I and II and of the energy comparison task in Tables III and IV. The data for the noncalculus physics students are contained in Tables I and III and the data for the honors students are in Tables II and IV. The columns in the tables are labeled by the three levels of investigator intervention: (i) no intervention, (ii) attention drawn to the starting procedures, and (iii) explicit mention

Table I. Results of the momentum comparison task in the noncalculus physics group ($N = 16$).

Level of intervention	Correct comparison	Comparison incorrect or inadequately justified	Indeterminate
Initial comparison: No interviewer intervention	0	100%	0
Comparison after discussion of starting conditions	0	81%	19%
Comparison after explicit discussion	6%	25%	69%

Table II. Results of the momentum comparison task in the honors calculus physics group ($N = 12$).

Level of intervention	Correct comparison	Comparison incorrect or inadequately justified	Indeterminate
Initial comparison: No interviewer intervention	25%	75%	0
Comparison after discussion of starting conditions	58%	25%	17%
Comparison after explicit discussion	67%	25%	8%

of impulse or work. The columns are labeled by a description of the adequacy of the comparisons made by the students. Comparisons have been divided into three groups: (i) correct comparisons, (ii) incorrect or inadequately justified comparisons, and (iii) indeterminate comparisons.

As can be seen from the data, a greater number of the honors students solved the energy comparison task than solved the momentum comparison task. Furthermore, those who compared the energies correctly were able to do so with less help from the investigator than was necessary for successful completion of the momentum comparison task. With intervention by the instructor, almost all the honors students were eventually able to conclude that the two pucks had equal kinetic energy. However, only about two-thirds were able to use sound physical reasoning to decide that the brass puck had the larger momentum.

Almost none of the noncalculus physics students was able to apply the concepts of impulse or work to a comparison of either the momenta or kinetic energies of the two pucks. Intervention by the investigator did not seem to help these students as it had the honors students. In fact, there was an increase in the number of "indeterminate" responses on both tasks after investigator intervention.

IV. REASONING ON THE TASKS BEFORE INVESTIGATOR INTERVENTION

At least as important as the correctness of the comparisons made by the students are the explanations they gave in support of their responses. The reasoning used reveals a great deal about the nature of the difficulties.

There were four possible responses on both the momentum and energy comparisons tasks. Students could say that the momentum or kinetic energy of the brass puck was larger or smaller than the momentum or kinetic energy of the plastic puck, or that the momenta or kinetic energies of both pucks were equal. The fourth possible response was that one could not tell from the given information whether there was a difference in these quantities. All four responses were obtained on each task.

A. Momentum comparison task

The data in Table V show the percentage of students in each group who gave each type of response on the momentum comparison task before any investigator intervention. The initial response of most students in both the honors

Table III. Results of the kinetic energy comparison task in the noncalculus physics group ($N = 16$).

Level of intervention	Correct comparison	Comparison incorrect or inadequately justified	Indeterminate
Initial comparison: No interviewer intervention	0	88%	13%
Comparison after discussion of starting conditions	0	69%	31%
Comparison after explicit discussion	0	69%	31%

Table IV. Results of the kinetic energy comparison task in the honors calculus physics group ($N = 12$).

Level of intervention	Correct comparison	Comparison incorrect or inadequately justified	Indeterminate
Initial comparison: No interviewer intervention	50%	50%	0
Comparison after discussion of starting conditions	75%	17%	8%
Comparison after explicit discussion	83%	8%	8%

Table V. Student comparisons of the momenta of the ice pucks before investigator intervention (numbers are percent of each sample). P_B = momentum of brass puck, P_P = momentum of plastic puck.

	Honors calculus physics ($N = 12$)	Noncalculus physics ($N = 16$)
$P_B = P_P$	58%	50%
$P_B > P_P$	25%	19%
$P_B < P_P$	0	19%
Cannot tell	17%	12%

and the noncalculus physics courses was that the momenta of the two pucks were equal. One-fourth of the honors students responded correctly that the brass puck had the greater momentum. None concluded that the brass puck had a smaller momentum than the plastic puck. However, the same number of noncalculus physics students said that the momentum of the brass puck was greater as said that it was smaller.

In Table VI are the reasoning schemes commonly used by students in justifying their comparisons and the percentages of students who gave these different types of explanations before investigator intervention. When Tables V and VI are examined together, it can be seen that students who made the proper comparison often did not arrive at this conclusion by correct reasoning. As stated earlier, unless the comparisons and reasoning were both correct, the response was not considered correct. Although almost 20% of the noncalculus physics students stated that the momentum of the brass puck was greater than that of the plastic puck, none was able to justify this response with an argument involving impulse. The lack of a correct explanation, together with the fact that an equal number of students made the converse claim, suggests that the noncalculus physics students who chose correctly were simply guessing.

Of the students in both groups who concluded incorrectly that the momenta were the same, by far the most common justification given was what might be described as a "compensation argument." The following excerpt is illustrative.

(I, investigator; S, student.)

I: Do the brass puck and plastic puck have the same momentum or different?

S: I think they have the same [momentum]...because, well, momentum is mass times velocity...so the brass puck has more mass, [and] a slower velocity...I'm not sure if they are exactly equal, but with the same force they should be equal because the smaller puck has less mass [and] a higher velocity.

This student reveals the key element of his analysis when he describes momentum as "mass times velocity." He reasons that the larger mass of the brass puck is probably compensated for by its lower velocity when the quantity mv is compared. The student confirms his belief in this analysis by going through the complementary argument for the plastic puck, i.e., he points out that the plastic puck has less mass but a larger speed. He further supports his conclusion that the momenta should be the same by noting that the two pucks are each subject to the same force.

The quality of the argument illustrated in the last interview excerpt indicates that the student has the requisite mathematical capability to deal with the material. It is clear, however, that he has not been able to connect force and time with change in momentum in a way that could be useful in analyzing the motion he has observed. The student seems to think of momentum simply in terms of a definition rather than as a concept that can be applied to account for what happens in the physical world.

The vast majority of students who stated that the momenta were equal justified their response with some variation of the compensation argument. There was one student in each group, however, who gave an explanation based solely on the equality of the forces on the brass and plastic pucks.

Although initially only 25% of the honors students stated that the brass puck had the larger momentum, all who made this correct comparison reasoned correctly that it received a larger impulse. Furthermore, as can be seen in Table II, many students in the honors course who initially used a compensation argument, were with the help of the investigator, eventually able to respond correctly. In contrast, even with assistance, the noncalculus physics students were not very successful in using an impulse argument in comparing the momenta.

B. Energy comparison task

The data in Table VII show the percentage of students who gave each type of response on the energy comparison task before any discussion with the investigator. Table VIII lists the reasoning schemes identified and the corresponding percentages of students who used them before the investigator intervened. About one-half of the honors students answered correctly that the kinetic energies of the two pucks were equal. Slightly fewer than one-third of the

Table VI. Common reasoning schemes on the momentum comparison task before investigator intervention (numbers are percent of each sample). P_B = momentum of brass puck, P_P = momentum of plastic puck.

Comparison	Explanation	Honors calculus physics ($N = 12$)	Noncalculus physics ($N = 16$)
$P_B > P_P$	Larger impulse received[a]	25%	0
$P_B = P_P$	Compensation argument	50%	44%
$P_B = P_P$	Equal applied force	8%	6%
No specific comparison	Confused discussion	17%	50%

[a] Correct response.

Table VII. Student comparisons of the kinetic energies of the ice pucks before investigator intervention (numbers are percent of each sample). T_B = kinetic energy of brass puck, T_P = kinetic energy of plastic puck.

	Honors calculus physics ($N = 12$)	Noncalculus physics ($N = 16$)
$T_P = T_B$	50%	31%
$T_P < T_B$	0	25%
$T_P > T_B$	33%	38%
Cannot tell	17%	6%

noncalculus physics students gave this response. As can be seen from a comparison of Tables VII and VIII, all of the honors physics students who stated that the kinetic energies were equal recognized that the work done on both pucks was the same, but none of the noncalculus physics students who made this choice used this type of argument. Furthermore, as Table III indicates, none of the noncalculus physics students was able to give the proper explanation even after a considerable amount of prompting by the investigator. It is interesting to note, both on this task and on the momentum comparison task, that if explanations had not been required, a significantly different impression of student understanding would have resulted.

As was the case in comparing the momenta, there appeared to be a systematic tendency to make one particular incorrect comparison of the kinetic energies. The initial response of about one-third of the honors students, and more than one-third of the noncalculus physics students, was that the plastic puck had greater kinetic energy than the brass puck. The same type of compensation argument students used to conclude falsely that the momenta were equal appeared to underlie the incorrect conclusion that the plastic puck had a greater kinetic energy. The following interview excerpt illustrates the reasoning typically used.

I: Do the brass puck and the plastic puck have the same kinetic energy or different?

S: I think the smaller puck would have a larger kinetic energy...because kinetic energy is $mv^2/2$ and since the v is squared, the one with the larger velocity would probably have a larger kinetic energy.

This student clearly exhibits some ability to reason mathematically. The argument that speed is a more important variable than mass in comparing kinetic energies, since speed appears quadratically rather than linearly in the definition, is a relatively sophisticated type of analysis. Again, as in the case of the momentum comparison task, the essential physics of the problem has been missed. The student's reasoning is based soley on the definition of kinetic energy and lacks any reference to the way in which the work done on the puck is related to the change in kinetic energy.

The reasoning used in the compensation argument is mathematically nontrivial, if physically inadequate. Among the noncalculus physics students, there were other examples of incorrect reasoning that were much less sophisticated. The following excerpt is one such example taken from a discussion after the investigator had intervened.

I: What does that term kinetic energy mean? What do you think of when you hear the term kinetic energy?

S: I think of the formula...isn't it one-half mass times velocity squared?

I: Yes, that's the definition of kinetic energy. So what does that imply about which one had the greatest kinetic energy?

S: Actually if I reasoned that the momenta are the same, then I would have to say the kinetic energies are the same because the quantities involved are the same ones.

I: Can you be more specific? How are you thinking here?

S: Well, they both incorporate mass and velocity.

This student had earlier defined momentum as mv and here defines kinetic energy as $mv^2/2$. The conclusion that the momenta and kinetic energies are equal seems based only on the idea that they are both combinations of mass and velocity. This level of reasoning was common in the noncalculus physics group, but was not encountered at all among the honors physics students.

V. REASONING ON THE TASKS AFTER INVESTIGATOR INTERVENTION

Students who failed to make a correct comparison on one or both of the tasks often initially used compensation arguments to justify their responses. After the investigator focused their attention on the starting conditions of the pucks and explicitly mentioned the terms "work" and "impulse," specific difficulties with these concepts began to emerge. The two excerpts below come from discussions that took place relatively late in two separate interviews. Although both focus on the energy comparison task, they illustrate the type of questioning and student response that occurred after investigator intervention.

In the first interview, the student failed to analyze the motion correctly even after being reminded to consider the

Table VIII. Common reasoning schemes on the kinetic energy comparison task before investigator intervention (numbers are percent of each sample). T_B = kinetic energy of brass puck, T_P = kinetic energy of plastic puck.

Comparison	Explanation	Honors calculus physics ($N = 12$)	Noncalculus physics ($N = 16$)
$T_P = T_B$	Same work done[a]	50%	0
$T_P > T_B$	Compensation argument	33%	25%
$T_P = T_B$	Equal applied force	0%	19%
No specific comparison	Confused discussion	17%	56%

[a] Correct response.

starting conditions. He is now being asked explicitly about the term "work."

I: Have you ever heard the term work? Do you remember what that word means in physics?

S: Work was...the change in kinetic energy...or, um, let me think here...I think it might have been the force times...I'm not sure, I think I recall the formula R, F, the cosine of the angle between the two. But we just did problems on that and I can't remember exactly.

This student remembers, or at least is able to repeat, some key ideas. He is even able to state that "work was...the change in kinetic energy." The understanding of the concept of work, however, seems to be limited to repeating the elements of a formula. He is not able to connect the symbols with the features of the demonstration. There is no evidence that he understands kinetic energy at that level either. Although he states that work is equivalent to the change in kinetic energy, he does not seem to understand the relationship at a level that would allow him to apply his knowledge to the task at hand. This type of response was fairly typical of students from the noncalculus physics class.

The next excerpt demonstrates that even when there was some understanding of work and kinetic energy considered separately, it did not follow that a student understood the connection between these concepts. The transcript comes from the portion of the interview in which the student was asked specifically if she was familiar with the term "work." As in the interview quoted above, this question was asked after explicit reference to the starting conditions had failed to elicit a proper comparison of kinetic energies.

I: What ideas do you have about the term work?

S: Well, the definition that they give you is that it is the amount of force applied times the distance.

I: Okay. Is that related at all to what we've seen here? How would you apply that to what we've seen here?

S: Well, you do a certain amount of work on it for the distance between the two green lines: You are applying a force for that distance, and after that point it's going at a constant velocity with no forces acting on it.

I: Okay, so do we do the same amount of work on the two pucks or different?

S: We do the same amount.

I: Does that help us decide about the kinetic energy or the momentum?

S: Well, work equals the change in kinetic energy, so you are going from zero kinetic energy to a certain amount afterwards...so work is done on each one...but the velocities and the masses are different so they [the kinetic energies] are not necessarily the same.

In her first three statements this student indicates a satisfactory understanding of the concept of work. She associates the applied force and the distance over which it acts and points out that in the demonstration the same amount of work is done on each puck. In her last statement she mentions that the work done equals the change in kinetic energy. Nevertheless, she still is unable to conclude that the kinetic energies must be the same. In other words, although there appears to be a satisfactory understanding of work and an ability to state the work-energy theorem, the student seems to be distracted by the observed dissimilarity of the masses and speeds.

It should be noted that had the interview been terminated any earlier than it was, the impression would have been that the student's understanding was adequate. After all, almost everything said was correct. It was only by continuing to probe her responses that the investigator was able to determine that the student did not actually make the connection between the work-energy theorem and the moving pucks. Unlike a physicist, the student did not see the demonstration in terms of a direct application of the formula to the real world.

VI. CONCLUSION

Many of the students who participated in this study demonstrated by their performance in introductory physics that they were able to answer examination questions covering the relevant course material. The honors students, in particular, seemed to have little trouble in applying the concepts of impulse, momentum, work, and energy in the solution of some rather sophisticated problems. Yet, almost all of the students in the noncalculus physics course and many of the honors students experienced considerable difficulty in a straightforward application of the impulse-momentum and work-energy theorems to the actual one-dimensional motion of an object under a constant force.

A. Discussion of results

It is evident from the discussion of student performance during the interviews that success or failure on the impulse-momentum and work-energy tasks requires more than memorization of the relevant theorems and the definitions of the quantities that are involved. To be able to apply these relationships to real world situations requires knowledge at a deeper level.

In order to analyze the demonstration correctly, the student must understand the operational definitions for work and impulse. He must further recognize that the definition of kinetic energy and momentum as particular functions of v is not an arbitrary choice. The changes in these quantities are related in a very precise way to the integrals over time and distance of the net force applied on a body. It is because of their connection to impulse and work that the quantities mv and $\frac{1}{2}mv^2$ have special significance. Moreover, it is important that students understand that the impulse-momentum and work-energy theorems express physical laws that relate two different, precisely defined quantities. Moreover, in each case there is a cause and effect relationship. This subtle, but nevertheless, critical point seemed to elude many of the participants in the study. It was apparent during the interviews that students often thought of the symbol $=$ as representing simply a mathematical identity with no dynamic quality.[4] For example, they did not interpret the work-energy theorem as a statement that doing work on a body produces an increase in its kinetic energy.

To make a correct comparison of the observed motions, the raw intuition that pushing on an object makes it go faster is not sufficient. Both the brass puck and the plastic puck visibly increase in speed during the demonstration and changes in both kinetic energy and momentum result from this increase. It is, in fact, this similarity between momentum and kinetic energy for motion in one dimension that requires a comparison task to help make clear the distinction.

B. Implications for instruction

The results of this investigation showed that many of the participants in the study had failed to recognize the significance of the impulse-momentum and work-energy theorems. Most of the noncalculus physics students seemed to have emerged from their course unable to recognize the critical elements in either of these relations. (There are, of course, more subtle aspects of interpretation that this research did not address.[5]) There is no reason to believe that the instruction these students received was inferior to the usual presentation of this, or any other, topic in introductory physics.

It has been our experience that fundamentally important features of concepts that are not easily visualized will be missed if they are presented only verbally, whether by textbook or in lecture. The impulse-momentum and work-energy theorems are a case in point. The demonstrations used in the comparison tasks provide a particularly simple set of circumstances in which these relations can be readily applied. Yet most of the participants in the study failed to make the appropriate connection. Incorporated into instruction, either as demonstrations or as problems, tasks such as those discussed in this paper can help provide the practice needed in applying the impulse-momentum and work-energy theorems to real world events.

The presentation of both theorems may suffer from premature emphasis on their application to systems of objects for which momentum or kinetic energy is conserved. A deeper understanding of these relations may result if their application in single particle dynamics is stressed. Before encountering the conservation laws, students should be given many opportunities to use the impulse-momentum and work-energy theorems to find the kinematical and dynamical quantities that they have previously obtained through the application of Newton's laws and the kinematical equations. For example, the final speed or duration of motion for a single body under the influence of an external force is readily found from impulse and momentum considerations. Exposure to this alternative approach can help students recognize that the theorems have a meaning that is independent of their role in the derivation of the conservation laws.

To develop a functional understanding, most students need experience in interpreting the formal relationships of physics in a variety of different contexts and under different conditions. A deep conceptual understanding is unlikely to be achieved, however, if students passively observe the instructor perform a demonstration or work a problem. In our own teaching, we have found it necessary to engage students actively in making explicit the connections between the algebraic formalism and real world applications.

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[4] A discussion of difficulties in interpreting the equality sign appears in A. B. Arons, *Handbook of Introductory Physics Teaching* (unpublished), Chap. 3.

[5] For a discussion of complications involved in applying the work-energy theorem in situations that are less idealized than the demonstration used in this study, see B. A. Sherwood, Am. J. Phys. **51**, 597 (1983).

Solutions to Laplace's equation using spreadsheets on a personal computer

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Using a spreadsheet routine on a personal computer, we obtain solutions to Laplace's equation quickly, even with dielectric interfaces present. This provides an alternative to solving electrostatics problems normally dealt with by relaxation methods in FORTRAN programs or Fourier series solutions.

I. INTRODUCTION

We outline a numerical technique for solving several classes of electrostatics problems using Laplace's equation, the personal computer, and a "spreadsheet" program. In particular the problems of interest are one-, two-, and three-dimensional systems with either one or two dielectrics present, and we seek solutions in charge-free regions.