

DENOTE YOUR ANSWERS CLEARLY. PLEASE²!

Question 1 [10 points]. A household is endowed with \bar{L} units of time which they can spend on either consumption (c) or leisure (L). The preferences are captured by the following utility function:

$$U(c, L) = c^{\frac{3}{5}} L^{\frac{2}{5}}$$

The household earns a wage w for each hour of work (l).

(a) What is the household labor supply l without any taxation?

(b) Suppose the government uses a *progressive linear income tax* to raise revenue and total tax liability of the household is $T = t(Y - E)$ where Y is income, $t > 0$ is the marginal tax rate and $E > 0$ is the income exemption limit.

What is the household's budget constraint and labor supply?

(c) Suppose the government uses a *progressive wage tax*. The total tax liability of the household is $T = c(w - w_e)l$, where $c > 0$ is a constant and $w_e > 0$ is a break-even wage: those with $w > w_e$ pay the wage tax and those with $w < w_e$ receives a wage subsidy.

What is the household's budget constraint and labor supply?

Question 2 [10 points]. Robinson Crusoe lives on an uninhabited island. His only source of food is coconuts (c), and he can collect 20 coconuts in an hour (h). Thus, his utility depends only on the number of coconuts he consumes and the number of hours that he spends collecting coconuts during a month.

$$u = c - h^2$$

(a) Write down Robinson's budget constraint? What are the optimal choices of coconuts and hours of work (c^* , h^*) that maximize Robinson's utility. What is his maximum utility?

(b) After 8 years a rescue ship arrives! However, it is just a British navy frigate that declares the island to be a British colony. Now, instead of being rescued Robinson has to pay the British Queen 32 coconuts every month. (She really loves coconuts apparently!) What is Robinson's new budget constraint? What is his optimal choice of c and h now? What is his utility?

(c) After 2 years the fixed tax of 32 coconuts is replaced with an income tax. The British now demand $\frac{1}{5}$ of all the coconuts collected by Robinson. What is Robinson's budget constraint? What is his optimal choice of c and h now? How much tax does Robinson pay?

How many coconuts would have to be given to him to make him as well off as he was under the fixed tax? What is the welfare cost of the income tax, measured in coconuts?

Question 3 [10 points]. Let's evaluate whether tax revenue can increase with a decrease in the tax rate. Suppose that individual preferences over consumption (c) and leisure (L) can be represented by a utility function

$$u(c, L) = c + 400 \ln L.$$

The hourly wage is \$25 and people have up to 60 hours per week to devote for work hours.

(a) Derive the expressions for optimal leisure (L), labor supply (l) and consumption (c). What is the optimal number of hours worked and consumption without taxation?

(b) The government introduces a linear income tax t that applies to wage. What are the optimal leisure hours (L), labor supply (l), consumption (c), and government revenue $R(t)$?

(c) What is the Laffer Curve equation here? The government is trying to maximize tax revenue. It would like a tax rate of 75%, but officials do not know how to take derivatives and FOC maximization problems to check if this is the revenue maximizing rate.

Therefore, they asked you to calculate revenue possibilities and labor supply hours for each of the following tax rates: 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%. Which tax rate would you recommend to collect maximum revenue?

Can you make a rough plot of the Laffer curve for a given tax rate?

Suppose that individual preferences over consumption (c) and leisure (L) take the Cobb-Douglas form instead. Thus,

$$u(c, L) = c^\alpha L^{1-\alpha}$$

where $\alpha = \frac{2}{3}$.

(d) What is the hours of work (l) function with the tax rate t now?

(e) What is the Laffer Curve equation now? Again, calculate revenue possibilities and labor supply hours for each of the following tax rates: 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%. Which tax rate would you recommend to collect maximum revenue now?

Question 4 [10 points]. The economy consists of 2 consumers with skill levels s_1 and s_2 , such that $s_2 > s_1$. Allocations to the low skilled (high skilled) consumer 1 (2) are then $x_1, z_1, (x_2, z_2)$.

(a) For the utility function

$$U = u(x) - \frac{z}{s}$$

show that incentive compatibility requires that

$$z_2 = z_1 + s_2[u(x_2) - u(x_1)]$$

(b) For the utilitarian social welfare function

$$W = u(x_1) - \frac{z_1}{s_1} + u(x_2) - \frac{z_2}{s_2}$$

express W as a function of x_1 and x_2 only.

(c) Set $u(x_i) = \ln x_i$ for $i = 1, 2$ and derive the *optimal* values of x_1 and x_2 and z_1 and z_2 .

Question 5 [10 points]. Residents of a city vote directly over the level of public spending. There are 3005 voters in the city. Voters differ in their income y : the mean income of the voters is \$50,000 and the median income is \$40,000. Each voter has same preferences, represented by the utility function

$$U(x, G) = \ln x + \ln G$$

where x is the residents' after-tax income and G is the total level of public spending. Public spending is financed by a *head tax* t .

- Set up the voter's utility in terms of the tax rate being chosen. Are voter's preferences single peaked?
- If the Median Voter Theorem applies, what tax *level* is the Condorcet winner?
- Suppose now that the level of spending G is being voted on. What level of public spending would be chosen according to majority rule?

Question 6 [10 points]. You have been called upon by the Minister of Finance to provide a succinct 2 page memo on tax policy recommendation. Read the following article and provide a summary of the main points, results and lessons on income tax policy design.

Diamond, Peter, and Emmanuel Saez. 2011. "The Case for a Progressive Tax: From Basic Research to Policy Recommendations." *Journal of Economic Perspectives*, 25(4): 165-90.

Papers in this Journal are written by top economists (P. Diamond won the Nobel prize) for a broad, general audience. They are never technical, but are rich in key, important ideas and new findings.

Yes, we are economist, but that does not mean we don't care about good writing and clear communication of key ideas. Your 2 page memo should be typed in 12pt font, double spaced and 1 inch margins. You can go beyond two pages if you have "something good to say" or to complete a thought, but try to keep it within 2 pages. This has to be in your words and **without** any use of quotations from the papers.

The usual "gimmicks" of using 1.5 inch margins, spreading the text around to 'fill the page' and reach the full 2 pages or read just the intro and conclusion will not get you the desired points. It will only make me unimpressed and deduct points for attempting to insult my intelligence. I think you are all better than this.

BONUS Question [6 points]. The above Diamond-Saez paper is actually a sort of response to the tax policy recommendations of

Mankiw, N. Gregory, Matthew Weinzierl, and Danny Yagan. 2009. “Optimal Taxation in Theory and Practice.” *Journal of Economic Perspectives*, 23(4): 147-74.

Read the Mankiw, Weinzierl, Yagan article and provide the same 2 page summary of the main points, results and lessons on income tax policy design. Just like for Diamond-Saez.

[HINT: Mankiw & et al. argue for some different policy recommendations, especially about income taxes. Diamond-Saez, in their appendix, compare their lessons and indicate where they disagree with Mankiw & et al.. You should read those few appendix pages as well. (Not the online appendix, but the one that comes with the paper right away.)]