

Homework 3 due June 9 (within 10 minutes after the class ends)

Total points: 100

Student's Name:

Please show your work only whenever it is indicated and make sure your writing is clear. Attach all the sheets with a stapler.

Problem 1 (45 points):

There are two goods X and Y and two consumers named Ann and Bob. $U_A(X_A, Y_A) = X_A Y_A$ and $U_B(X_B, Y_B) = X_B Y_B^\alpha$ where $\alpha > 0$. $MU_{A,X} = Y_A$, $MU_{A,Y} = X_A$, $MU_{B,X} = Y_B^\alpha$, $MU_{B,Y} = \alpha X_B Y_B^{\alpha-1}$. Total endowments of goods X and Y are respectively denoted w_X and w_Y .

a) (10 points) Find Pareto efficient allocation in which $X_A = X_B = w_X/2$ (that is, find Y_A^* and Y_B^* such that, $X_A = X_B = w_X/2$ along with Y_A^* and Y_B^* constitutes PE allocation). Show your work.

b) (3 points) Is Y_A^* increasing or decreasing in w_Y ?

c) (3 points) Is Y_A^* increasing or decreasing in α ?

d) (3 points) Is Y_B^* increasing or decreasing in w_Y ?

e) (5 points) Is Y_B^* increasing or decreasing in α ?

f) (2 points) Interpret your answers to parts c and e.

Now for the rest of the parts suppose that $w_{A,X} = 1, w_{A,Y} = 1, w_{B,X} = 1, w_{B,Y} = 1$.

g) (4 points) What is total endowment of good X in the society?

h) (15 points) Normalize $p_X = 1$ and assume $\alpha = 4$. Solve for the general equilibrium price and quantities consumed. That is, find $p_Y, X_A^*, X_B^*, Y_A^*, Y_B^*$. Show your work.

Problem 2 (30 points):

Suppose $U_A(X, Y) = \min\{X, Y\}$ and $U_B(X, Y) = \min\{2X, Y\}$. $w_X = 1, w_Y = 1$.

a) (15 points) Draw the edgeworth box, label it completely and depict the set of Pareto efficient allocations.

For the following part, suppose that initial endowments are $w_{A,X} = 0.3, w_{A,Y} = 0.5, w_{B,X} = 0.7, w_{B,Y} = 0.5$.

b) (15 points) Which bundles are Pareto optimal if trade takes place and Ann obtains all the gains from trade?

Problem 3 (25 points):

Suppose Ann and Bob each have 40 units of good X and 40 units of good Y . $U_A(X, Y) = 1.5X + 1.8Y, U_B(X, Y) = 1.5X + 1.8Y$. $MU_{A,X} = MU_{B,X} = 1.5$ and $MU_{A,Y} = MU_{B,Y} = 1.8$.

a) (10 points) Are there Pareto improving trades? If yes, what are they?

Now suppose that Ann and Bob get contradictory information. Ann now thinks that good Y is totally worthless and Bob thinks that X is totally worthless. Hence, their utilities become $U_A(X, Y) = X, U_B(X, Y) = Y$.

b) (10 points) Draw the edgeworth box with indifference curves at the initial endowments. Clearly label everything.

c) (5 points) Assume Ann and Bob exhaust all potential Pareto improving trades. Which allocation(s) could result?