MAB127/MAB122/MAN122
Summer Semester 2011
Problem Solving Task 2

Due:	Thursday February 9, 2012 at 4:00pm (Week 11)
Submit to:	MAB127 Assignment Box on Level 6 of O Block
Weighting:	10%

Instructions

Your assignment may be neatly hand–written, or you may submit a printout of a document you have word processed.

Show full working.

The completed Assignment Cover Sheet (available on Blackboard) must be the first page of your submission.

Submit your assignment with a single staple in the upper left corner. Do **NOT** use folders, envelopes, plastic sleeves, etc.

Do **NOT** submit via Assignment Minder or your assignment **WILL NOT BE MARKED**.

Total of 60 marks, scaled to give a final percentage out of 10%.

Question 1.

Evaluate the double integral

$$\int_1^2 \int_0^{\frac{1}{x}} xy \, dy \, dx$$

(a) using the given order of integration; (2 marks)

(b) by reversing the order of integration. (4 marks)

Question 2.

Formulate a triple integral to compute the mass of an object which in the shape of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $z = 6 - 3x - 2y$ and whose density is given by the function $f(x, y, z)$. You do not need to actually evaluate the integral. (3 marks)

Question 3.

A particle, initially at rest, undergoes acceleration according to the vector valued function $\mathbf{a}(t) = \mathbf{i} - \mathbf{j} + 2t\mathbf{k}$ between $t = 0$ and $t = 10$. Determine the:

(a) displacement of the particle; (3 marks)

(b) distance travelled by the particle. (4 marks)

Question 4.

(a) Find the parametric equations for the line passing through the points $(1, 0, -2)$ and $(-1, 1, 0)$. (3 marks)

(b) Find the cartesian equation for the plane passing through the points $(1, -1, 0)$, $(1, 0, 2)$ and $(0, -2, 1)$. (5 marks)

(c) Find the point at which the line from (a) intersects the plane from (b). (4 marks)

Question 5.

Look up, figure out, or make an intelligent guess at the product rule for the scalar product. That is, a rule of the form

$$\frac{d}{dt}\left[\mathbf{a}(t) \cdot \mathbf{b}(t)\right] = ? + ?$$

Verify your proposed rule on the functions (4 marks)

$$\mathbf{a}(t) = t\mathbf{i} + \sin(t)\mathbf{j} - e^t\mathbf{k} \text{ and } \mathbf{b}(t) = \cos(t)\mathbf{i} - t^2\mathbf{j} + e^{-t}\mathbf{k}.$$

Question 6.

It can be shown that the area of the surface described by the vector valued function $\mathbf{r}(s,t)$ between the limits $a \leq s \leq b$ and $c \leq t \leq d$ is given by

$$A = \int_a^b \int_c^d \left\| \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right\| \, dt \, ds.$$

Find the surface area of the bowl described by (7 marks)

$$\mathbf{r}(s,t) = s\cos(t)\mathbf{i} + s\sin(t)\mathbf{j} + s^2\mathbf{k}, \quad 0 \leq s \leq 1, \quad 0 \leq t \leq 2\pi.$$

Question 7.

Maxwell has come to you for assistance. The torrential rain from the recent weather has caused significant damage to the courtyard of his house by washing away a substantial amount of soil. He has given you a diagram of his backyard, and has approximated the edges of the area needing repair. He knows that the concrete from his verandah and the garden edge are 10cm above ground level. The soil needs to be filled in so that it is at this constant height of 10cm is across the entire courdyard. He estimates that the height of the soil remaining after the deluge can be modelled by the function $f(x,y) = -\frac{1}{2}xy$, measured in metres. What is the volume of soil required to repair the courtyard? (7 marks)

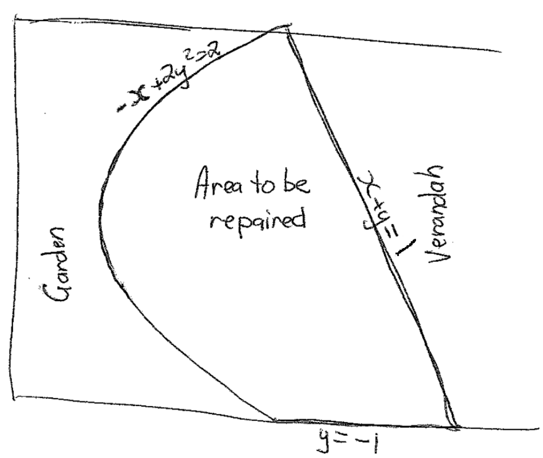

Figure 1: Diagram of Maxwell's courtyard

Question 8.

A particle undergoing motion has position vector $\mathbf{r}(t)$ given by

$$\mathbf{r}(t) = R\cos(\psi)\sin(t)\mathbf{i} + R\sin(\psi)\sin(t)\mathbf{j} + R\cos(t)\mathbf{k}$$

where t is time and R, ψ are positive constants.

(a) Show that the particle's velocity vector is orthogonal to its position for all time. (2 marks)

(b) Show that the particle's acceleration vector is parallel and opposite to its position vector for all time. (2 marks)

(c) Show that the particle is travelling at constant speed. (2 marks)

(d) Look up the definition of *angular momentum* using the internet or a reference book, and verify that the angular momentum of the particle is constant for all time. (3 marks)

Legibility, setting out, communication and submitted in correct format (5 marks)