# Homework Sheet 4
# Due: July 7, 2017

The following questions are from chapter 6 of the OpenStax Calculus textbook. The problem number from the book is listed in parenthesis.

1. (4) True or false: If $\sum_{n=1}^{\infty} a_n x^n$ has radius of convergence $R > 0$ and if $|b_n| \leq |a_n|$ for all $n$, then the radius of convergence of $\sum_{n=1}^{\infty} b_n x^n$ is greater than or equal to $R$. (Hint: consider The Direct Comparison Test.)

2. (8) Suppose that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$. Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{a_n x^n}{2^n}$.

3. (66) Use partial fractions to find the power series of the function

$$\frac{3}{(x+2)(x-1)}.$$

4. (92) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

by calculating $f'(1/3)$, where $f(x) = \sum_{n=0}^{\infty} x^n$. (Your answer needs to be a *number*, not an infinite sum.)

5. (118) Find $T_2(x)$ (the quadratic Taylor polynomial) for the function $f(x) = \cos(2x)$ centered at $\pi$.

6. (142) Find the Taylor series for $\sin x$ centered at $\pi$.

7. (214) Find the Maclaurin series of

$$F(x) = \int_0^x \frac{\sin t}{t}\, dt$$

by integrating

$$\frac{\sin t}{t} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} t^{2n}$$

term by term.

# Additional Problems

**6.1 Power Series and Functions** 5, 2, 15, 19, 25

**6.2 Properties of Power Series** 64, 70, 80, 102

**6.3 Taylor and Maclaurin Series** 130, 146, 154, 158

**6.4 Working with Taylor Series** 203, 209, 220, 235