

Math 1P97. Assignment 5 (4%).

Due date: Monday, June 26, 6:45 pm

Student's name:

Box #:

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Student's name: _____

ID: _____

Total mark: _____

NOTE: Students are expected to complete all questions on the assignment. However, only a subset of questions will be considered for marking. Marks will be deducted for incomplete assignments.

1. Solve the differential equation $y' = 4e^{2x} + 6x^2$ with the initial condition $y(0) = 4$.

2. Find the indefinite integrals

a) $\int \frac{2x - 8}{x^2 - 8x + 16} dx =$

b) $\int \frac{2x}{(x^2 + 3)^3} dx =$

$$\text{c) } \int 2xe^{x^2+5} dx =$$

$$\text{d) } \int \sqrt[5]{7x+12} dx =$$

$$\text{e) } \int e^{8x-20} dx =$$

$$\text{f) } \int \frac{(\ln x)^2}{x} dx =$$

3. According to a model for predicting the height of preschool children, the rate of growth of a typical child is

$$R'(t) = 26e^{-t} + 6.3, \quad \left(\frac{1}{4} \leq t \leq 6\right)$$

centimeters per year, where t is measured in years. The height of a typical 3-month old preschool child is 60 cm.

- a) Find a function $R(t)$ for predicting the height of a typical preschool child at age t .

- b) Estimate the height of a typical 3-yr-old child.

4. Evaluate the definite integral.

a) $\int_0^1 (4x^3 - 6x^2 + 3\sqrt{x} + 1) \, dx =$

b) $\int_{-1}^1 (x^2 + 1)^2 \, dx =$

c) $\int_0^1 6x (x^2 + 9)^2 dx =$

d) $\int_{-2}^0 e^{2x+4} dx =$

e) $\int_1^e \frac{(3 \ln x)^2}{x} dx =$

f) $\int_{-2}^{-1} \frac{4x^3 - 3x^2 + 1}{x^2} dx =$

5. The rate at which the risk of Down syndrome is changing is approximated by the function

$$r'(x) = 0.004641x^2 - 0.3012x + 4.9, \quad (20 \leq x \leq 45)$$

where $r'(x)$ is measured in percentage of all births/year and x is the maternal age at delivery.

a) Find a function f giving the risk as a percentage of all births when the maternal age at delivery is x years, given that the risk of Down syndrome at age 30 is 0.14% of all births

b) Based on this model, what is the risk of Down syndrome when the maternal age at delivery is 25 years?

- c) What is the estimated average of the risk of Down syndrome when the maternal age at delivery is between 25 and 35 years?

(*Hint:* The average value (AV) of f over $[a, b]$ is $AV = \frac{1}{b-a} \int_a^b f(x) dx$)

6. (Maple Question) Find the area of the region enclosed by the following graphs.

To find the area between a function f and g , use the command

```
[> int(f-g,x=a..b);
```

- a) $y = x^2 - 3x - 10$, $y = -x^2 + x + 20$
b) $xy = 9$, $x + y = -10$

Submit a printout of your Maple worksheet.

7. (Maple Question) Plot the graphs of the functions of $f = e^{x^2-3x-4}$ and $g = -x^2 + \frac{15}{4}x$ and find the area of the region totally enclosed by the graphs of these functions using Maple.

- a) Write your name, restart Maple and load the plot package

```
[> #Q7. <name> <student number>
```

```
[> restart:with(plots);
```

- b) Define the function $f(x)$:

```
[> f:=exp(x^2-3*x-4);
```

- c) Define the function $g(x)$:

```
[> g:=-x^2+15*x/4);
```

- d) Find the intersections of these graphs:

```
[> a:=fsolve(f=g,x=0..1);
```

```
[> b:=fsolve(f=g,x=3..4);
```

- e) plot both functions f and g on the same graph for x :

```
[> plot([f,g],x=a..b,color=[blue,green],thickness=2);
```

- f) Find the exact area between $f(x)$ and $g(x)$ on the interval $[a, b]$:

```
[> S:=int(g-f,x=a..b);
```

- g) Submit a printout of your Maple worksheet