Name_____ Date_____

How Far Will You Travel? Worksheet

PRECALCULUS: VECTORS

Directions: Suppose that you plan to take a trip to your dream destination. You would like to know the shortest distance between your starting point and your destination. When calculating distances on a plane, you need only consider two dimensions because you are on a flat surface. However, when finding distances between two points on Earth, you must take into the account the curvature of a sphere. In this portfolio, you will extend your knowledge of two-dimensional vectors to three-dimensional vectors in order to find the shortest distance between two points on the surface of Earth.

Part 1

Choose your starting location and a dream destination location and find the latitude and longitude for each.

- Round to four decimal places.
- Use a negative sign for southern latitudes and/or western longitudes, and use a positive sign for northern latitudes and/or eastern longitudes.

Name of Location	Latitude	Longitude

Cite your source where you found the latitude and longitude for both locations:

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Every point on Earth can be represented by a three-dimensional vector. The vector's starting point is at the center of Earth.

To calculate the unit vectors corresponding to each of your locations, apply these formulas:

$\mathbf{v} = \langle x, y, z \rangle$, where the following applies:

$x = \cos(\text{latitude}) \cdot \cos(\text{longitude})$

$y = \cos(\text{latitude}) \cdot \sin(\text{longitude})$

$z = \sin(\text{latitude})$

Calculate the corresponding three-dimensional unit vector for each of your locations.

Name of Location	***x*-coordinate**	***y*-coordinate**	***z*-coordinate**	$\langle x, y, z \rangle$

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Part 2

In order to find the distance between these two locations, you need to know the angle between the two vectors (or the central angle). To find the angle, you will need to find the dot product of your two vectors and find the magnitude of each.

Finding the dot product of 3D vectors is essentially the same as finding the dot product of 2D vectors:

$$\langle a,b,c\rangle \bullet \langle d,e,f\rangle = ad+be+cf$$

Finding the magnitude of 3D vectors is also essentially the same as it is for 2D vectors:

$$\|\langle a,b,c\rangle\| = \sqrt{a^2+b^2+c^2}$$

Now find the angle θ between your two vectors using this formula:

$$\theta = \cos^{-1}\left(\frac{\mathbf{v}\bullet\mathbf{w}}{\|\mathbf{v}\|\|\mathbf{w}\|}\right)$$

Show your work in the space below. Be sure to find θ in radians, not in degrees:

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Part 3

Although Earth is not a perfect sphere, assume that it is for the purposes of this portfolio and use the formula for the arc length of a segment of a great circle on a sphere, $s = r\theta$, to find the distance between your two locations. In this case, r is the radius of Earth, which is approximately 3,963.2 miles. Find the distance between your two locations.

Show your work in the space below.

What is the distance between your two locations?

Part 4

Write a paragraph that discusses what you learned about vectors from this portfolio.

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