

Math 1P97. Assignment 3 (4%).

Due date: Monday, June 5, 6:45pm

Student's name:

ID:

Box #:

Student's name: _____

Total mark: _____

NOTE: Students are expected to complete ALL questions on the assignment. However, only a subset of questions will be considered for marking. Marks will be deducted for incomplete assignments. **Show all your work.**

1. (Marginal cost, revenue, and profit) The quantity demanded each day of the Walter Serkin recording of Beethovens Moonlight Sonata, manufactured by Phonola Record Industries, is related to the price/compact disc. The equation

$$p = -0.38x + 60.4 \quad (0 \leq x \leq 100)$$

where p denotes the unit price in dollars and x is the number of discs demanded, relates the demand to the price. The total monthly cost (in dollars) for pressing and packaging x copies of this classical recording is given by

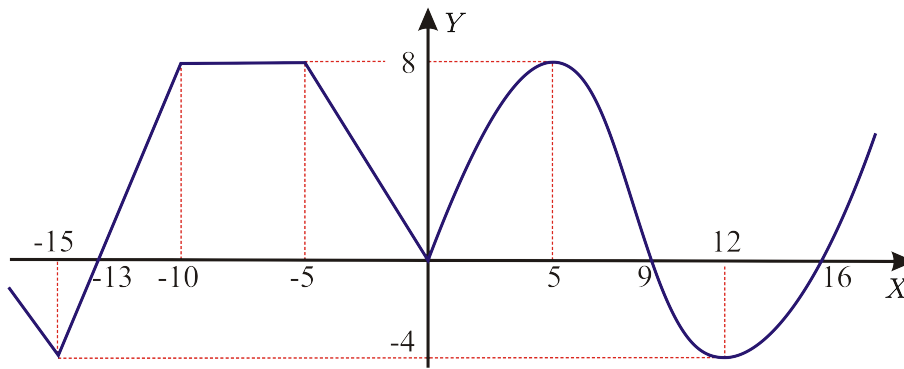
$$C(x) = 20 + 0.4x - 0.005x^2 \quad (0 \leq x \leq 100)$$

- a) Find the revenue function $R = R(x)$ and the profit function $P = P(x)$.

- b) Find the marginal profit function.

- c) How many copies should Phonola produce each month to maximize the profit?

2. Refer to the following graph of a function $y = f(x)$:



- a) find the points where $f(x) = 0$
- b) find the intervals where $f'(x) > 0$
- c) find the intervals where $f'(x) < 0$
- d) find the points/intervals where $f'(x) = 0$
- e) find the points where $f'(x)$ doesn't exist.
- f) find the points/intervals where $f''(x) = 0$.
- j) find the intervals where $f''(x) > 0$.
- h) find the intervals where $f''(x) < 0$.

3. Find the interval(s) where the function $f(x)$ is increasing and the interval(s) where it is decreasing.

a) $f(x) = x^3 + 3x^2 - 9x + 10$

b) $f(x) = x(x^2 - x(x - 1) + 1)$

c) $f(x) = (4 - x^2)^2$

4. Find the relative maxima and relative minima, if any, of each function.

a) $f(x) = x^3 - 3x^2 - 9x + 10$

b) $f(x) = x^4 - 8x^2$

c) $f(x) = (x^2 + 4x + 4)^2$

5. Find the absolute maximum value and the absolute minimum value of each function.

a) $f(x) = -x^2 - x + 6$ on $[-1, 3]$

b) $f(x) = x\sqrt{16 - x^2}$ on $[-4, 0]$

6. (Price-Demand) Suppose the quantity demanded weekly of the Super Titan radial tires is related to its unit price by the equation

$$p + x^2 = 144$$

where p is measured in dollars and x is measured in units of a thousand. How fast is the quantity demanded changing when $p = 63$, and the price/tire is increasing at the rate of \$2/week?

7. (Maximizing yield) An apple orchard has an average yield of 36 bushels of apples/tree if tree density is 22 trees/acre. For each unit increase in tree density, the yield decreases by 2 bushels/tree. How many trees should be planted in order to maximize the yield?

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8. (Minimizing packaging costs) A rectangular box is to have a square base and a volume of 40 ft^3 . If the material for the base costs 0.3\$/square foot, the material for the sides costs 0.1\$/square foot, and the material for the top costs 0.2\$/square foot, determine the dimensions of the box that can be constructed at minimum cost.

9. Find the horizontal and vertical asymptotes of the graph of the function.

a) $f(x) = \frac{1+x}{1-x}$

b) $f(x) = \frac{x}{x^2-1}$

c) $f(x) = \frac{x^3-3x}{(x-1)(x^2+2)}$