Unit 2 Sample Work Problems

pg 423 - 426 #'s 7, 13, 17, 19, 21, 23, 27, 31, 33, 37, 41, 51, 53, 57, 61, 65, 67, 73, and 81

6-1 Roots and Radical Expressions

Quick Review

You can simplify a radical expression by finding the roots. The **principal root** of a number with two real roots is the positive root. The principal ***n*th root** of b is written as $\sqrt[n]{b}$, where b is the **radicand** and n is the **index** of the radical expression.

For any real number a, $\sqrt[n]{a^n} = \begin{cases} a \text{ if } n \text{ is odd} \\ |a| \text{ if } n \text{ is even} \end{cases}$.

Example

What is the simplified form of $\sqrt{36x^6}$?

$\sqrt{6^2x^6}$	Find the root of the integer.
$= \sqrt{6^2(x^3)^2}$	Find the root of the variable.
$= 6\|x^3\|$	Take the square root of each term. Since the index is even, include the absolute value symbol to ensure that the root is positive even when x^3 is negative.

Exercises

Find each real root.

5. $\sqrt{25}$
6. $\sqrt{0.49}$
7. $\sqrt[3]{-8}$
8. $-\sqrt[3]{8}$

Simplify each radical expression. Use absolute value symbols when needed.

9. $\sqrt{81x^2}$
10. $\sqrt[3]{64x^6}$
11. $\sqrt[4]{16x^{12}}$
12. $\sqrt[5]{0.00032x^5}$
13. $\sqrt{\frac{9x^4}{36}}$
14. $\sqrt[3]{125x^6y^9}$

6-2 Multiplying and Dividing Radical Expressions

Quick Review

If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real numbers, then

$(\sqrt[n]{a})(\sqrt[n]{b}) = \sqrt[n]{ab}$, and, if $b \neq 0$, then $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$.

To **rationalize the denominator** of an expression, rewrite it so that the denominator contains no radical expressions.

Example

What is the simplest form of $\sqrt{32x^2y} \cdot \sqrt{18xy^3}$?

$\sqrt{(32x^2y)(18xy^3)}$	Combine terms.
$= \sqrt{(4^2 \cdot 2x^2y)(3^2 \cdot 2xy^3)}$	Factor.
$= \sqrt{4^2 \cdot 3^2 \cdot 2^2x^3y^4}$	Consolidate like terms.
$= \sqrt{4^2 \cdot 3^2 \cdot 2^2(x^2x)(y^2)^2}$	Identify perfect squares.
$= 4 \cdot 3 \cdot 2xy^2\sqrt{x} = 24xy^2\sqrt{x}$	Extract perfect squares.

Exercises

Multiply if possible. Then simplify.

15. $\sqrt[3]{9} \cdot \sqrt[3]{3}$
16. $\sqrt[3]{-7} \cdot \sqrt[3]{49}$
17. $\sqrt{2} \cdot \sqrt{8}$

Multiply and simplify.

18. $\sqrt{8x^2} \cdot \sqrt{2x^2}$
19. $5\sqrt[3]{9y^2} \cdot \sqrt[3]{24y}$

Divide and simplify.

20. $\sqrt{\frac{128}{8}}$
21. $\frac{\sqrt[3]{81x^5y^3}}{\sqrt[3]{3x^2}}$
22. $\frac{\sqrt[4]{162x^4}}{\sqrt[4]{2y^8}}$

Divide. Rationalize all denominators.

23. $\frac{\sqrt{8}}{\sqrt{6}}$
24. $\frac{\sqrt{3x^5}}{8x^2}$
25. $\frac{\sqrt[3]{6x^2y^4}}{2\sqrt[3]{5x^7y}}$

6-3 Binomial Radical Expressions

Quick Review

Like radicals have the same index and the same radicand. Use the distributive property to add and subtract them. Use the FOIL method to multiply binomial radical expressions. To rationalize a denominator that is a square root binomial, multiply the numerator and denominator by the conjugate of the denominator.

Example

What is the simplified form of $\sqrt{18} + \sqrt{50} - \sqrt{8}$?

$\sqrt{18} + \sqrt{50} - \sqrt{8}$

$= \sqrt{3^2 \cdot 2} + \sqrt{5^2 \cdot 2} - \sqrt{2^2 \cdot 2}$ Factor.

$= 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2}$ Simplify each radical.

$= (3 + 5 - 2)\sqrt{2}$ Combine like terms.

$= 6\sqrt{2}$ Simplify.

Exercises

Add or subtract if possible.

26. $10\sqrt{27} - 4\sqrt{12}$

27. $3\sqrt{20x} + 8\sqrt{45x} - 4\sqrt{5x}$

28. $\sqrt[3]{54x^3} - \sqrt[3]{16x^3}$

Multiply.

29. $(3 + \sqrt{2})(4 + \sqrt{2})$

30. $(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11})$

31. $(10 + \sqrt{6})(10 - \sqrt{3})$

Divide. Rationalize all denominators.

32. $\frac{2 + \sqrt{5}}{\sqrt{5}}$

33. $\frac{3 + \sqrt{18}}{1 + \sqrt{8}}$

6-4 Rational Exponents

Quick Review

You can rewrite a radical expression with a rational exponent. By definition, if the nth root of a is a real number and m is an integer, then $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$; if m is negative then $a \neq 0$. Rational exponents can be used to simplify radical expressions.

Example

Multiply and simplify $\sqrt{x}\left(\sqrt[4]{x^3}\right)$.

$\sqrt{x}\left(\sqrt[4]{x^3}\right) = x^{\frac{1}{2}} \cdot x^{\frac{3}{4}}$ Rewrite with rational exponents.

$= x^{\frac{5}{4}}$ Combine exponents.

$= \sqrt[4]{x^5}$ Rewrite as a radical expression.

Exercises

Simplify each expression.

34. $25^{\frac{1}{2}}$

35. $81^{\frac{1}{4}}$

36. $16^{\frac{1}{3}} \cdot 4^{\frac{1}{3}}$

37. $5^{\frac{3}{2}} \cdot 5^{\frac{1}{2}}$

Write each expression in simplest form.

38. $\left(x^{\frac{1}{4}}\right)^4$

39. $\left(-8y^9\right)^{\frac{1}{3}}$

40. $\left(\sqrt{9xy^2}\right)^4$

41. $\left(x^{\frac{1}{6}} y^{\frac{1}{3}}\right)^{-18}$

42. $\left(\frac{x^4}{x^{-1}}\right)^{-\frac{1}{5}}$

43. $\left(\frac{x^{\frac{1}{3}}}{y^{-\frac{2}{3}}}\right)^9$

6-5 Solving Square Root and Other Radical Equations

Quick Review

To solve a **radical equation**, you must isolate a radical expression on one side of the equation. You can then rewrite the radical expression using a rational exponent and use the reciprocal of the exponent to solve the equation.

For example, to solve a square root equation, you square each side of the equation. Check all possible solutions in the original equation to eliminate extraneous solutions.

Example

What is the solution of $4(x-2)^{\frac{2}{3}} = 16$?

$(x-2)^{\frac{2}{3}} = 4$	Isolate the radical.
$((x-2)^{\frac{2}{3}})^{\frac{3}{2}} = 4^{\frac{3}{2}}$	Raise both sides to the $\frac{3}{2}$ power.
$(x-2)^{\frac{6}{6}} = 4^{\frac{3}{2}}$	Law of exponents.
$\lvert x-2 \rvert = 8$	Simplify.
$x = 10$ or $x = -6$	Solve for x.

Exercises

Solve each equation. Check for extraneous solutions.

44. $2 + \sqrt{x+5} = 4$

45. $3\sqrt{2x+6} = 18$

46. $5(3x+1)^{\frac{1}{4}} = 10$

47. $4(3x-3)^{\frac{2}{3}} = 36$

48. $\sqrt{3x+3} - 1 = x$

49. $\sqrt{x+6} + 2 = x + 6$

50. $\sqrt{5x+1} - 2\sqrt{x} = 1$

51. $\sqrt{2x+9} - \sqrt{x} = 3$

52. Electricity The power P, in watts, that a circular solar cell produces and the radius of the cell r in centimeters are related by the square root equation $r = \sqrt{\frac{P}{0.02\pi}}$. About how much power is produced by a cell with a radius of 12 cm?

6-6 Function Operations

Quick Review

When performing function operations, you can use the same rules you used for real numbers, but you must take into consideration the domain and range of each function. The composition of function g with function f is defined as $(g \circ f)(x) = g(f(x))$.

Example

Let $f(x) = x + 3$ and $g(x) = x^2 - 2$. What is $(g \circ f)(-2)$?

$g(f(-2)) = g((-2) + 3)$	Evaluate $f(-2)$.
$= g(1)$	Simplify.
$= (1)^2 - 2$	Evaluate $g(f(-2))$.
$= -1$	Simplify.

Therefore, $(g \circ f)(-2) = -1$

Exercises

Let $f(x) = x - 4$ and $g(x) = x^2 - 16$. Perform each function operation and then find the domain.

53. $f(x) + g(x)$

54. $g(x) - f(x)$

55. $f(x) \cdot g(x)$

56. $\frac{g(x)}{f(x)}$

Let $g(x) = 5x - 2$ and $h(x) = x^2 + 1$. Find the value of each expression.

57. $(h \circ g)(-1)$

58. $(h \circ g)(0)$

59. $(g \circ h)(2)$

60. $(g \circ h)(a)$

61. Discounts A grocery store is offering a 50% discount off a \$4.00 box of cereal. You also have a \$1.00 off coupon for the same cereal. Use a composite function to show whether it is better to use the coupon before or after the store discount.

6-7 Inverse Relations and Functions

Quick Review

If a relation or a function is described by an equation in x and y, you can interchange x and y to get the inverse. The domain of a function becomes the range of its inverse, and the range of a function becomes the domain of its inverse.

Example

What is the inverse of $f(x) = \sqrt{x - 10}$?

$y = \sqrt{x - 10}$	Rewrite using y.
$x = \sqrt{y - 10}$	Interchange the x and y values.
$x^2 = y - 10$	Square each side.
$y = x^2 + 10$	Solve for y.
$f^{-1}(x) = x^2 + 10$	Write the inverse function.

The domain of $f(x)$ is $x \geq 10$, which means the range of $f^{-1}(x)$ is $y \geq 10$. Also, since the range of $f(x)$ is $y \geq 0$, the domain of $f^{-1}(x)$ is $x \geq 0$.

Exercises

Find the inverse of each function. Determine whether each inverse is a function.

62. $f(x) = 2x^2 - 8$

63. $f(x) = 15 - 3x$

64. $f(x) = \sqrt{x + 6}$

65. $f(x) = (2x - 3)^2$

Graph each function and its inverse. Describe the domain and range of each.

66. $f(x) = 4x - 1$

67. $f(x) = (x + 3)^2$

68. $f(x) = \sqrt{x - 3}$

69. $f(x) = 6 - 5x^2$

70. **Geometry** The volume of cube is determined by the formula $V = s^3$, where s is the length of one side. Find the inverse formula. Use it to find the side length of a cube with a volume of 64 ft^3.

6-8 Graphing Radical Functions

Quick Review

The function $f(x) = \sqrt{x}$ is the parent function of the **square root function** $f(x) = a\sqrt{x - h} + k$. The graph of $f(x) = a\sqrt{x}$ is a stretch $(a > 1)$ or a shrink $(0 < a < 1)$ of the parent function. The graph of $f(x) = a\sqrt{x - h} + k$ is a translation h units horizontally and k units vertically of $y = a\sqrt{x}$. The graph of $f(x) = \sqrt[n]{x}$ is transformed by a, h, and k in the same way as the graph of $f(x) = \sqrt{x}$.

Example

Describe the graph of $y = \sqrt{4x + 12}$.

$y = \sqrt{4x + 12}$	
$y = \sqrt{4(x + 3)}$	Factor the polynomial.
$y = 2\sqrt{x + 3}$	Simplify the radical.

The graph of $y = \sqrt{4x + 12}$ is the graph of $y = 2\sqrt{x}$ translated 3 units to the left.

Exercises

Graph each function. Find the domain and range.

71. $y = \sqrt{x} - 5$

72. $y = \sqrt{x + 8}$

73. $y = 5\sqrt{x} + 9$

74. $y = -\sqrt{x - 4}$

75. $y = \sqrt[3]{x + 10}$

76. $y = -\sqrt[3]{x - 2} + 5$

Rewrite each function to make it easy to graph using transformations. Describe each graph.

77. $y = \sqrt{9x - 27} + 4$

78. $y = -3\sqrt{4x - 16}$

79. $y = \sqrt[3]{8x + 24}$

80. $y = \sqrt{\frac{x - 4}{4}} + 6$

Solve each equation by graphing.

81. $5 = -\sqrt{x - 3}$

82. $\sqrt{8x - 16} = 2\sqrt{x + 2}$