

Derivatives and Optimization

(Math 157; TBA #2)

Please work the problems on your own paper, and double check to make sure that numeric answers contain the correct units. The due date is Tuesday, July 25, but don't wait too long to get started, or you will not have time to ask questions.

1. The demand equation, $p = \frac{50}{\sqrt{x}}$; $1 \leq x \leq 30$, gives the price, p , at which x toasters can be sold.
 - (a) Find $\frac{dx}{dp}$ when $x = 16$ toasters and explain what it means in the context of this application. Be specific with respect to values and units.
 - (b) Find $\frac{dR}{dp}$ when $x = 16$ toasters and explain what it means in the context of this application. Be specific with respect to values and units.
2. The cost and revenue for the production and sale of x megatons of Luvitup are given by $C(x) = 5,000 + x(x - 30)^2$; $0 \leq x \leq 10$ and $R(x) = -300x^2 + 4,000x$ respectively.
 - (a) Describe how you would use calculus to find the maximum possible profit.
 - (b) Use calculus to find the maximum possible profit (to the nearest dollar). If you wish, you may use your calculator instead of algebra to find zeros of derivatives, but your solution must involve calculus.
 - (c) You can see that $R''(x) < 0$ for all values of x . What is the significance of this fact as it pertains to:
 - i. The graph of the revenue function?
 - ii. The marginal revenue?
 - (d) **(Bonus)** What price should we charge for a megaton of Luvitup in order to maximize profit? Make your answer accurate to the nearest penny.
3. The average annual interest rate, $r\%$, for commercial bank credit cards from the year 2000 through the year 2005 can be modeled by the equation $r = \sqrt{-1.7409t^4 + 18.07t^3 - 52.68t^2 + 10.9t + 249}$ where t represents the year, with $t = 0$ corresponding to the year 2000. Check to see that you understand how to use this model correctly by verifying that the average annual rate in the year 2000 was 15.78% when rounded to the nearest hundredth of a percent. Answer each of the following using the proper units and with percentages rounded to the nearest hundredth of a percent.
 - (a) According to this model, at what rate was the average annual interest rate changing at the beginning of 2002 (i.e. when $t = 2$)?
 - (b) During what month of what year was the average annual interest rate decreasing the fastest, and what was the rate of decrease at that time? For this problem, you may use calculus, algebra, and/or your graphing calculator in any way you see fit, but your answers must have the desired level of accuracy, and you must show or explain how you did it.