Math 150A - Section 4.5 - Optimization Problems

Approaching Optimization Problems:

- Draw a picture, labeling all variables. Write down the information given.
- Find an equation for the quantity Q that is to be maximized or minimized in terms of the other variables.
- If there is more than one independent variable, find additional equations. Use these new equations to express Q in terms of only one variable.
- Differentiate this equation with respect to the independent variable.
- Optimize using the methods of sections 4.1 and 4.3.

Example: Rectangles

Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.

Example: Inscribing a Rectangle in a Triangle

Find the area of the largest rectangle that can be inscribed in a right triangle with legs of lengths 3 cm and 4 cm if two sides of the rectangle lie along the legs.

Example: Making a Box

A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

Example: Distance to a Curve

Find the point on the curve $y = \sqrt{x}$ that is closest to the point (3,0).

Distance Formula:

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

First Derivative Test for Absolute Extreme Values

Suppose c is a critical number of a continuous function f defined on an interval.

(a) If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum of f.

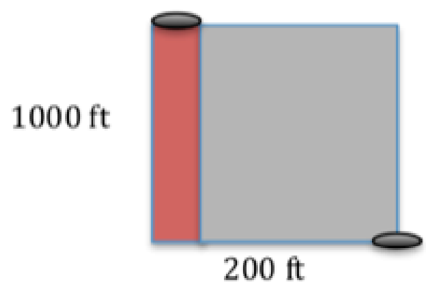
(b) If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum of f.

Example: The Best Coffee Can

A coffee company is making cylindrical cans to fill with 40 in^3 of a new drink. What dimensions of the can will minimize the cost of the metal to manufacture the can?

Example: Running to Class

You are late to your calculus class, and you must run across campus. In particular, you need to go 1000 ft South and 200 ft East to get to McCarthy Hall. If you travel along the main walk-way (shown in red), then you can walk at 10 ft/s. However, if you cut across the quad (grey), you can only walk at 3 ft/s. How far down the main walk-way should you travel before cutting across the quad in order to minimize your time to class?



Example: Inscribing a Rectangle in a Semicircle

Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

Applications to Business and Economics

Let $C(x)$ be the cost of producing x items. Then the **marginal cost** is $C'(x)$.

Let $p(x)$ be the price per unit if selling x items. This is the **price function** or the **demand function**. Then the revenue for selling x items at a price $p(x)$ is given by the **revenue function**

$$R(x) = xp(x).$$

The **marginal revenue** is then given by $R'(x)$. The **profit function** is

$$P(x) = R(x) - C(x),$$

and the **marginal profit** is $P'(x)$.

Example: Blu-ray Players

An electronic store has been selling 100 Blu-ray players per week at \$300 each. A market survey showed that if the price was reduced by \$15, then the number of units sold would increase by 30. Find the demand function and revenue function. How should the store price the device to maximize revenue?

Example: Pumpkins

A garden has 200 pounds of pumpkins growing. Every day, the total weight of the pumpkins increases by 5 pounds, while the price per pound goes down \$0.01. If the price is currently \$0.90/lb, how much longer should the gardener wait to sell the pumpkins in order to obtain the highest possible price?

Example: Coffee Maker Production

A company is producing coffee makers, and the weekly cost to produce x coffee machines is given by

$$C(x) = 100 + 150x + 0.5x^2, \qquad 0 \leq x \leq 100$$

and the price per coffee maker at production level x is $p(x) = 250 - 1.5x$. Find the weekly production level that produces a maximum profit.

Example: Laying a Pipeline

A company is planning on connecting an oil refinery to oil storage tanks through a pipeline. It costs \$40,000 per km over land and \$50,000 per km underwater to lay this pipeline. The oil refinery is on the east bank of a north flowing river that is 2 km wide. The storage tanks are 3 km further south on the west bank. What is the minimum possible cost?
