Linear Algebra
3rd Quiz

Notation.

- P_n = space of polynomials, with real coefficients, of degree at most n.
- $\mathbb{R}^{m\times n}$ = space of m by n real matrices.

Problem 1 Find out which of the following transformations are linear and for those that are linear, determine whether they are isomorphisms.

1. $T : P_2 \to \mathbb{R}, \quad T(f(t)) = f(0).$
2. $T : \mathbb{C} \to \mathbb{C}, \quad T(x + iy) = x - iy.$
3. $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}, \quad T(M) = M^2.$

Problem 2 Consider the (standard) basis $\mathcal{B}_1 = \{1, x, x^2, x^3, x^4\}$ of P_4.

1. Prove that the set $\mathcal{B}_2 = \{x^4, 2x^3, 1 - x^2, 3x - 1, 2x\}$ is a basis of P_4. Find the change of basis matrix S from $\mathcal{B}_1$ to $\mathcal{B}_2$.
2. Let $T : P_4 \to P_4$ be the linear transformation defined by

$$T(p(x)) = p''(x) + p'(x) + p(x).$$

   Find the matrix of T with respect to $\mathcal{B}_2$.

Problem 3 Given the subspace of $\mathbb{R}^{2\times 2}$

$$S = \{\begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \mid \begin{bmatrix} 1 & -\frac{4}{3} \end{bmatrix} \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 0\},$$

find its dimension and a basis $\mathcal{B}$ of S such that $[A]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, for $A = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$.